Jitter Analysis of QPSK and BOC(n,n) GNSS Signals

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Biography

Bilal Amin was born in Rawalpindi, Pakistan, in 1982. He received the BE and MS degrees from the National University of Sciences and Technology (NUST), Pakistan and University of New South Wales (UNSW), Australia in 2003 and 2004 respectively. He is currently working towards his PhD in the School of Surveying and Spatial Information Systems, University of New South Wales. His research interests include Software Radio (SDR) based Global Navigation Satellite Systems (GNSS) receiver front-end design, jitter analysis of GNSS signals, Analog to Digital Convertors (ADCs) and sampling techniques for multiple GNSS signals.

Abstract

This paper analyzes the effects of jitter on the GPS and Galileo navigation signals. Jitter effects have usually been modelled as additive noise, based on a sinusoidal input signal, and limits the achievable Signal-to-Noise Ratio (SNR). Analysis shows that psec-level jitter specifications are required in order to keep jitter noise well below the thermal noise for software radio satellite navigation receivers. However, analysis of a BPSK system shows that large errors occur if the jittered sample crosses a data bit boundary. Quadrature Phase Shift Keying (QPSK) modulation achieves greater bandwidth efficiency than BPSK. The new GPS L5 signal is QPSK modulated. The Binary Offset Carrier (BOC) spreading modulation has recently been recommended by GPS-Galileo Working group on Interoperability and Compatibility for adoption by Europe's Galileo program for its Open Service (OS) signal at the L1 frequency and by the United Sates for its modernized GPS L1 Civil (L1C) signal. BOC signals have more transitions and hence jitter creates more woes. The aim of this paper is to derive expressions for noise due to jitter taking into account the transitions probability in QPSK and BOC systems. Both simulations and analysis are used to give a better understanding of jitter effects on Software Radio GNSS receivers.

1 Introduction

Aperture jitter effects in sampled systems have usually been modeled as additive noise, based on sinusoidal input signal. However, in PSK systems i.e. BPSK and QPSK, large errors can occur if the jitter causes a sample to appear on the wrong side of a data bit boundary. In a simple PSK system, the carrier has constant amplitude and the phase of signal can change. These jumps in phase in general cause instantaneous jumps in voltage. So if a sampling system should ideally sample before such a jump but instead samples after, then the error in the sample amplitude will in general be greater than if the same process had occurred on a sinusoidal signal. Quadrature-phase-shift-keying (QPSK) modulation is an effective means of achieving high bandwidth efficiency in wired or wireless communications systems. The new GPS L5 signal is QPSK modulated, centered on 1176.45 MHz. Its two components have each a different spreading code at chipping frequency of 10.23 MHz. Although QPSK can be viewed as a quaternary modulation, it is easier to see it as two independently modulated quadrature carriers. With this interpretation, the even (or odd) bits are used to modulate the in-phase component of the carrier, while the odd (or even) bits are used to modulate the quadrature-phase component of the carrier. BPSK is used on both carriers and they can be independently demodulated. The carrier for legacy GPS civil signal L1 at 1575.42 MHz, using Binary Phase Shift Keyed (BPSK) signal with a rectangular pulse shape and a spreading code chip rate of 1.023 MHz, denoted BPSK-R(1). Although very good performance can be obtained with the BPSK modulated signal, it has been recognized that navigation signals can achieve better multipath performance using spreading modulations that provide more power at high frequencies away from center frequency. Split spectrum signals in general, and Binary Offset Carrier (BOC) signals in particular, have their energy pushed away from the center towards NRZ nulls. GPS is taking advantage of this improved spectrum utilization in their modernized signals and several Galileo signals use variations of BOC modulation. BOC modulation is an extension of so-called Tricode Hexaphase Modulation, which is based on Manchester coding of each chip symbol.

Previous work has examined two formulae for jitter noise power. The first formula assumes that the jitter offset is much less than a period of the carrier [3]. The second formula makes no assumption about the size of τ_n, i.e. offset to the ideal sampling instant due to jitter [4]. Other simple non-sinusoidal signals such as square and triangular waves have also been examined with regard to their jitter noise power [5]. The analysis in [1] evaluated the jitter requirement such that the noise due to sampling jitter at the carrier frequencies of GNSS signal was $10dB$ less than the thermal noise. For all GNSS bands, this requirement was of the order of picoseconds. Dempster [2] has derived an expression for the noise due to aperture jitter in BPSK systems and examined it for GPS L5 signal which has carrier 1176.45 MHz and chipping rate 10.23 Mcps. The expression is only significantly different for very small $\sigma_\tau (< 10^{-12})$. At $\sigma_\tau = 10^{-12} sec$, it shows that jitter noise is in fact about $3dB$ higher than previous expressions for a typical satellite navigation signal.

The paper is organized as follows: in section II, the jitter expression for QPSK system is derived and evaluated. In section III jitter in BOC modulated systems is derived and evaluated. Section IV concludes the paper.

2 Jitter in QPSK System

A QPSK system can be modeled by two BPSK signals in quadrature. We can represent QPSK signal as

$$y(t) = \frac{A}{\sqrt{2}}[d_1(t)\cos(\omega_c t+\phi) + d_2(t)\sin(\omega_c t+\phi)] \quad (1)$$

where $d_i(t)(i=1,2..)$ is the data, taking values of -1 and 1. Without loss of generality, the phase ϕ will be set to zero in our calculations. Because it is arbitrary and would vary uniformly across the realizations, the ensemble average of any time varying sine and cosine in the following is zero. If the signal into analogue to digital converter is y(t), then the jitter noise can be modeled as:

$$\varepsilon_\tau(n) = y(t_n+\tau_n) - y(t_n) \quad (2)$$

The power of the jitter noise signal is

$$N_\tau = E[\varepsilon_\tau^2(n)] - E[\varepsilon_\tau(n)]^2 \quad (3)$$

As per [1], the second term in equation (3) is zero, i.e. the expected error due to jitter is zero because positive and negative errors of the same magnitude are equally likely (assuming that data bit transitions are not correlated with the carrier phase). So

$$N_\tau = E[\varepsilon_\tau^2(n)] \quad (4)$$

$$\begin{aligned} N_\tau = E[(&\frac{A}{\sqrt{2}}d_1(t_n+\tau_n)\cos\omega_c(t_n+\tau_n) \\ &+\frac{A}{\sqrt{2}}d_2(t_n+\tau_n)\sin\omega_c(t_n+\tau_n)) \\ &-(\frac{A}{\sqrt{2}}d_1(t_n)\cos\omega_c(t_n)+\frac{A}{\sqrt{2}}d_2(t_n) \\ &\sin\omega_c(t_n))]^2 \end{aligned} \quad (5)$$

The above equation can be represented as $(a-b)^2 = a^2+b^2-2ab$. Solving first for a^2

$$\begin{aligned} a^2 &= \frac{A^2}{2}d_1^2(t_n+\tau_n)\cos^2\omega_c(t_n+\tau_n) \\ &\quad +\frac{A^2}{2}d_2^2(t_n+\tau_n)\sin^2\omega_c(t_n+\tau_n)) \\ &\quad +2\frac{A}{\sqrt{2}}d_1(t_n+\tau_n)\cos\omega_c(t_n+\tau_n) \\ &\quad \frac{A}{\sqrt{2}}d_2(t_n+\tau n)\sin\omega_c(t_n+\tau_n)) \end{aligned}$$

As $d^2(t)=1$ at all times,

$$\begin{aligned} a^2 &= A^2[1/2+(d_1(t_n+\tau_n)d_2(t_n+\tau_n) \\ &\quad \cos\omega_c(t_n+\tau_n)\sin\omega_c(t_n+\tau_n))] \end{aligned}$$

$$E[a^2] = E[A^2[1/2+d_1(t_n+\tau_n)d_2(t_n+\tau_n)$$

$$\sin 2\omega_c(t_n+\tau_n)]]$$

$E[sin2\omega_c(t_n+\Delta t]=0$ as it is oscillatory and gives zero when average over a cycle. Also, $E[d_1(t_n)d_2(t_n)]=0$ as d_1 and d_2 are uncorrelated.

$$E[a^2] = \frac{A^2}{2} \tag{6}$$

Similarly by solving b^2 we get

$$b^2 = A^2[1/2+d_1(t_n)d_2(t_n)\sin 2\omega_c(t_n)]$$

$$E[b^2] = E[A^2[1/2+d_1(t_n)d_2(t_n)\sin 2\omega_c(t_n)]]$$

$$E[b^2] = \frac{A^2}{2} \tag{7}$$

Now consider the $2ab$ term

$$\begin{aligned}2ab = A^2[&d_1(t_n)d_1(t_n+\tau_n)\cos\omega_c(t_n)\\ &cos\omega_c(t_n+\tau_n)+d_1(t_n)d_2(t_n+\tau_n)\\ &\cos\omega_c(t_n)sin\omega_c(t_n+\tau_n)+d_1(t_n+\tau_n)\\ &d_2(t_n)\sin\omega_c(t_n)cos\omega_c(t_n+\tau_n)+d_2(t_n)\\ &d_2(t_n+\tau_n)\sin\omega_c(t_n)sin\omega_c(t_n+\tau_n)]\end{aligned}$$

As before $E[d_i(t_i)d_j(t_j)cosw_c(2t_n+\tau_n)]=0$ and $E[d_i(t_i)d_j(t_j)sinw_c(2t_n+\tau_n)]=0$. Also $E[d_i(t_i)d_j(t_j)]=0$. After applying the assumptions $E[2ab]$ becomes

$$\begin{aligned}E[2ab] = \frac{A^2}{2}&E[\cos\omega_c(\tau_n)\big(d_1(t_n)d_1(t_n+\tau_n)\\ &+d_2(t_n)d_2(t_n+\tau_n)\big)]\end{aligned} \tag{8}$$

Taking the terms in turn and acknowledging that the data product will be -1 across a data bit boundary,

$$\begin{aligned}E[d(t_n)d(t_n+\tau_n)] = E[&Pr(d(t_n)=d(t_n+\tau_n)\\ &-Pr(d(t_n)\neq d(t_n+\tau_n)]\end{aligned}$$

If the data is uncorrelated, there is likely to be one transition every two bits and the likelihood of that occurring between t_n and $(t_n+\tau_n)$ is:

$$Pr(d(t_n)\neq d(t_n+\tau_n) = \frac{|\tau_n|}{2T_d}$$

where T_d is data bit period. So

$$\begin{aligned}E[d(t_n)d(t_n+\tau_n)] &= E[1-\frac{|\tau_n|}{2T_d}-\frac{|\tau_n|}{2T_d}]\\ &= 1-\frac{E[|\tau_n|]}{T_d}\end{aligned} \tag{9}$$

The mean of half normal distribution where the full normal had the variance σ^2 is $\sqrt{\frac{2}{\pi}}\sigma$ so equation 9 becomes

$$E[d(t_n)d(t_n+\tau_n)] = 1-\sqrt{\frac{2}{\pi}}\frac{\sigma_\tau}{T_d} \tag{10}$$

Also, from [2]

$$E[\cos\omega_c(\tau_n)] = 1-\frac{\omega_c^2\sigma_\tau^2}{2} \tag{11}$$

Substituting (10) and (11) in (8)

$$\begin{aligned}E[2ab] = \frac{A^2}{2}\Bigg[&(1-\frac{\omega_c^2\sigma_\tau^2}{2})\big((1-\sqrt{\frac{2}{\pi}}\frac{\sigma_\tau}{T_d})\\ &+(1-\sqrt{\frac{2}{\pi}}\frac{\sigma_\tau}{T_d})\big)\Bigg]\end{aligned} \tag{12}$$

From (6), (7) and (12) in (5) we get

$$\begin{aligned}N_{\tau_{QPSK}} = A^2-\frac{A^2}{2}\Bigg[&(1-\frac{\omega_c^2\sigma_\tau^2}{2})\big((1-\sqrt{\frac{2}{\pi}}\frac{\sigma_\tau}{T_d})\\ &+(1-\sqrt{\frac{2}{\pi}}\frac{\sigma_\tau}{T_d})\big)\Bigg]\end{aligned}$$

$$N_{\tau_{QPSK}} \approx \left(\sqrt{\frac{2}{\pi}}\frac{\sigma_\tau}{T_d}+\frac{\omega_c^2\sigma_\tau^2}{2}\right)$$

From [1],

$$N_{\tau_{BPSK}} \approx \left(\sqrt{\frac{2}{\pi}}\frac{\sigma_\tau}{T_d}+\frac{\omega_c^2\sigma_\tau^2}{2}\right)$$

Therefore,

$$N_{\tau_{QPSK}} \approx N_{\tau_{BPSK}}$$

The power of jitter noise in QPSK system is same as the BPSK system. This is reasonable as QPSK signal is composed of in-phase and quadrature-components and we have assumed both the signals with same power. This shows that the use of QPSK is no worse than BPSK and the results in [1] still hold for GPS L5 signal. A range of σ_τ was examined for GPS L5 signal which has a carrier 1176.45 MHz and chipping rate 10.23 Mcps. The results are shown in Fig.1 and it can be seen that the QPSK noise power at $\sigma_\tau = 10^{-12}sec$ is same as the BPSK system and in fact about $3dB$ higher than the old expression would suggest, which is significant.

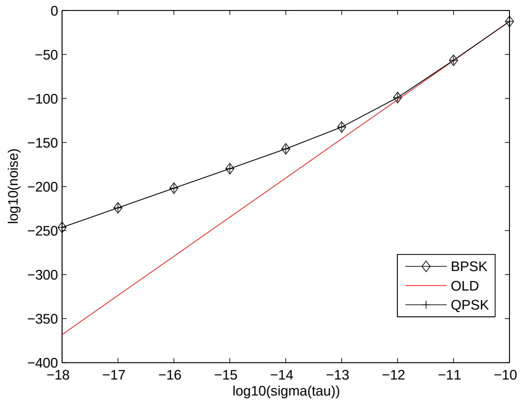

Figure 1. For the GPS L5 signal parameters and amplitude A=1, the comparison between BPSK, QPSK and sinusiodal expressions for the noise due to jitter.

3 Jitter in BOC Systems

BOC modulation is square subcarrier modulation, where the signal $d(t)$ is multiplied by a rectangular subcarrier of frequency f_{sub}, which splits the spectrum of the signal into parts.

Formally, the BOC-modulated signal is defined as

$$y(t) = A[d(t)\cos(\omega_c t + \phi) sgn[\sin(\omega_{sub} t + \phi)] \quad (13)$$

In the case of BOC modulation,

$$d_{BOC}(t) = d(t) sgn[\sin \omega_{sub} t]$$

So,

$$y(t_n) = A d_{BOC}(t_n)\cos(\omega_c t_n + \phi) \quad (14)$$

From (4)

$$N_\tau = E[(A d_{BOC}(t_n+\tau_n)\cos\omega_c(t_n+\tau_n) - (A d_{BOC}(t_n)\cos\omega_c(t_n)]^2$$

$$N_\tau = A^2 E[d_{BOC}^2(t_n+\tau_n)\cos^2\omega_c(t_n+\tau_n) - 2d_{BOC}(t_n)d_{BOC}(t_n+\tau_n)\cos\omega_c(t_n) \cos\omega_c(t_n+\tau_n) + d_{BOC}^2(t_n)\cos^2\omega_c(t_n)$$

Using the same reasoning as for BPSK,

$$N_\tau = A^2(1 - E[d_{BOC}(t_n)d_{BOC}(t_n+\tau_n)] \; E[\cos\omega_c\tau_n]) \quad (15)$$

Fig. 2 shows the timing relationships for BOC(n,n) signal. BOC(n,n) notation means a binary offset carrier with n being a 1.023 MHz square wave and m being a 1.023 MHz pseudorandom code. The BPSK code chips are $1/(n \times 1.023MHz)$ duration each and the square wave has a frequency of 1 cycle/chip. Therefore BOC(n,n) signal has a transition at the center of each code chip which makes it more vulnerable to sampling jitter. If the data is uncorrelated, there is likely to be three transitions every two bits and the likelihood of that occurring between t_n and $(t_n+\tau_n)$ is:

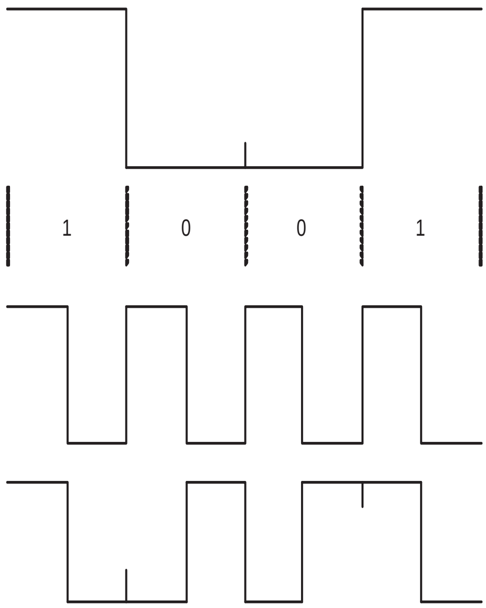

Figure 2. BOC(1,1) timing relationships a) data, b) offset carrier (square wave) c) BOC(n,n)

$$Pr(d_{BOC}(t_n) \neq d_{BOC}(t_n+\tau_n) = \frac{3|\tau_n|}{2T_d}$$

So,

$$E[d_{BOC}(t_n)d_{BOC}(t_n+\tau_n)] = 1 - \frac{3E[|\tau_n|]}{T_d} \quad (16)$$

$$E[d_{BOC}(t_n)d_{BOC}(t_n+\tau_n)] = 1 - 3\sqrt{\frac{2}{\pi}}\frac{\sigma_\tau}{T_d} \quad (17)$$

Also from [2]

$$E[\cos\omega_c(\tau_n)] = 1 - \frac{\omega_c^2\sigma_\tau^2}{2} \quad (18)$$

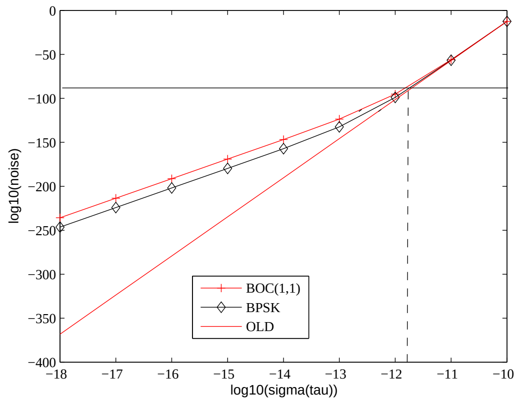

Figure 3. For the GPS L1 signal parameters and amplitude A=1, the comparison between BPSK and BOC expressions for the noise due to jitter.

Therefore from (17) and (18) in (15)

$$N_{\tau_{BOC}} = A^2\left(1 - \left(1 - 3\sqrt{\frac{2}{\pi}}\frac{\sigma_\tau}{T_d}\right)\left(1 - \frac{\omega_c^2\sigma_\tau^2}{2}\right)\right)$$

$$N_{\tau_{BOC}} = A^2\left(3\sqrt{\frac{2}{\pi}}\frac{\sigma_\tau}{T_d} + \frac{\omega_c^2\sigma_\tau^2}{2}\right) \qquad (19)$$

which completes the derivation for power of jitter noise in BOC(1,1) system. BOC(1,1) is the current baseline for Galileo's L1 signal [6], which has a carrier 1575.42 MHz and chipping rate 1.023 Mcps. The results are shown in Fig.3 and it can be seen that the BOC(1,1) noise power at $\sigma_\tau = 10^{-12}sec$ is $3dB$ higher than the BPSK system, which is significant. The jitter noise power for the BOC(1,1) system is worse than QPSK system because it has more transitions i.e. three transitions every two bits. For the Galileo L1 signal exploiting BOC(1,1) modulation in order to keep the jitter noise $10dB$ down from thermal noise $N_\tau < -97dBm$ as shown in Fig.3. The method in [7] is used to calculate the minimal allowable aperture jitter i.e. $2.5ps$, in case of BOC(1,1) modulation. The minimal allowable aperture jitter for BPSK and sinusiodal is $2.01ps$ and $2.0ps$ respectively. Therefore, in this case, using the new analysis has not required a major revision of jitter specification. However, in other Galileo signals (e.g. the BOC(15, 10) signal on E5 and BOC(15,2.5) signal on E1), it is expected that a similar analysis will be significant. Fig. 4 outlines the effect of analyzed jitter on the effectiveness of location estimation. As we have assumed that the jitter noise is well $20dB$ below the thermal noise, therefore the position error would remain constant as the jitter increases.

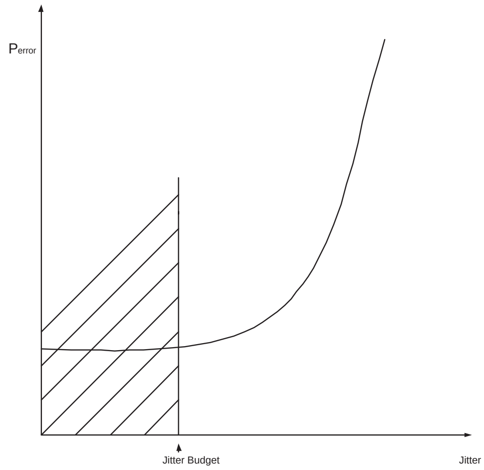

Figure 4. Effect of analyzed jitter on the effectiveness of location estimation

4 Conclusion

A new expression for noise due to jitter when sampling QPSK for software radio is derived and found to be identical to BPSK. Another expression for BOC(1,1) was also derived and an allowable jitter limit is calculated that satisfies the assumption.

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