Comparing the Quality Indicators of GPS Carrier Phase Observations

Chalermchon Satirapod
Jinling Wang

School of Geomatic Engineering
The University of New South Wales
Sydney NSW 2052 Australia
email: c.satirapod@student.unsw.edu.au
jinling.wang@unsw.edu.au

ABSTRACT

To achieve reliable GPS positioning results a realistic stochastic model for GPS carrier phase observations has to be specified for data processing. In order to develop such a stochastic model, the quality characteristics of GPS carrier phase measurements made by a receiver must be well understood. Recently investigators have used two types of data, Signal-to-Noise Ratio (SNR) and satellite elevation, as quality indicators for GPS carrier phase observations. This paper compares these two classes of data quality indicators. Single-differenced residuals are used to analyse the validity of the quality indicators, on a satellite-by-satellite basis. The results from a series of tests are presented and discussed.

1. INTRODUCTION

For all high-accuracy positioning applications, GPS carrier phase observations have to be used in the data processing step. It is well known that there are two important aspects to the optimal processing of GPS observations, the definitions of the so-called *functional model* and the corresponding *stochastic model*. The functional model describes the mathematical relationship between the GPS observations and the unknown parameters, while the stochastic model describes the statistics of the GPS observations (Leick, 1995). The present data processing models have been essentially unchanged since the early 1980s and are well understood. Data differencing techniques are extensively used for constructing the functional model as they can eliminate many of the troublesome GPS biases, such as the atmospheric bias, the receiver clock bias, the satellite clock bias, and so on. However, some unmodelled (or 'residual') biases still remain in the GPS observables following such differencing. In principle it is therefore possible to further improve the accuracy and reliability of GPS results through an enhancement of the stochastic model. The challenge is to find a way to realistically incorporate information on such unmodelled biases into the stochastic model.

Many researchers have emphasised the importance of the stochastic model especially for high-accuracy applications (for example, Barnes et al., 1998; Han, 1997; Satirapod, 1999; Wang, 1998; Wang et. al., 1998). Recently there has been interest in using two types of data, Signal-to-Noise Ratio (SNR) and satellite elevation, as quality indicators for GPS observations (for example, Gerdan, 1995; Han, 1997; Hartinger & Brunner, 1998; Jin, 1996; Lau & Mok, 1999; Talbot, 1988). It is important that a better understanding of these quality indicators is gained in order that they may be used correctly.

In this paper the quality indicators for GPS carrier phase observations are described, as well as the methodology used to assess them. A series of tests are described and the analysis of the results are then discussed. Finally, some conclusions are drawn based on the analysis of the data collected during the experiments.

2. QUALITY INDICATORS

2.1 Signal-to-Noise Ratio (SNR)

SNR has been widely used to construct a multipath mitigation model, as well as the stochastic model for high-accuracy applications. Most of the SNR models were designed to mitigate the multipath effect as multipath is a major concern in GPS positioning, especially in urban areas. For instance, the relationship between multipath and SNR or Carrier-to-Noise density ratio (C/No) has been investigated by many authors (for example, Comp & Axelrad, 1996; Lau & Mok, 1999; Sleewaegen, 1997; Talbot, 1988). More recently, SNR was introduced as a quality indicator for GPS observations and used to construct the stochastic model. In Spilker (1996), the relationship between the RMS phase noise (s_f) and the SNR_L is given as:

$$s_f^2 \cong \frac{1}{SNR_L} \qquad (1)$$

Langley (1997) claims that C/No is the key parameter in analysing GPS receiver performance and that it directly affects the precision of GPS observations. Hartinger & Brunner (1998) also stated that SNR data directly expresses the quality of the individual GPS phase values, and the performance of their SIGMA-ε model is demonstrated according to the following formula:

$$s_1^2 = S_1 \bullet 10^{\frac{-C/No}{10}} \qquad (2)$$

where the subscript indicates the L1 signal and S_1 consists of the carrier loop noise bandwidth and a conversion term from cycle^2 to mm^2. From an analysis of many data sets the value of S_1 was estimated to be about 1.6×10^4. Lau & Mok (1999) described the performance of the Signal-to-noise ratio Weighted Ambiguity function Technique (SWAT), where they emphasised the close relationship between the SNR cofactor matrix and the elevation angle (as SNR is almost directly proportional to the elevation angle in 'not-too-noisy environments').

2.2 Satellite Elevation Angle

Satellite elevation angle information is often used to construct a modified stochastic model. Jin (1996) stated that the accuracy of GPS code observations at comparatively low satellite elevation angles decreases with decreasing elevation, and that the relationship can be modelled quite well by an exponential function. He described a relationship between satellite elevation and RMS error, which may be expressed as:

$$y = a_0 + a_1 \bullet \exp\{-x/x_0\} \qquad (3)$$

where y is the RMS error, a_0, a_1 and x_0 are coefficients dependent on the receiver brand and the observation type, and x is the satellite elevation angle in degrees. This relationship has been used by many researchers in various GPS data processing scehemes (for example, Gerdan, 1995; Han, 1997; Jin, 1996; Wang, 1999; Barnes et al., 1998). It is likely that the satellite elevation angle information is now used as a quality indicator of GPS observations in many scientific and commercial GPS software packages.

2.3 Single-Differenced Model

The single-differenced model (between receivers) is chosen as the method of analysis for this study since the validity of the two above-mentioned quality indicators can be assessed on a satellite-by-satellite basis. For short baselines, the single-differenced model can be expressed as (Hofmann-Wellenhof et al., 1998; Leick, 1995; Teunissen & Kleusberg, 1996):

$$\Phi^{j}_{AB}(t) = \frac{1}{l}\, r^{j}_{AB}(t) + N^{j}_{AB} - f\, d_{AB}(t) + e^{j}_{AB}(t) \tag{4}$$

where the superscript *j* denotes the satellite, the subscripts *A* and *B* indicate the two receivers, the index *t* denotes the epoch at which the data were collected, Φ is the measured carrier phase, l is the wavelength of carrier phase, r is the distance to the satellite, *N* is the single-differenced integer ambiguity, *f* denotes the frequency of the satellite signal and d is the relative receiver clock bias. The term *e* represents all remaining errors, including random noises of receivers and systematic errors, such as unmodelled multipath effects, atmospheric delay, etc.

In order to compute the single-differenced residuals, the double-differenced ambiguities have to be resolved to their integer values correctly. This procedure is performed by the standard GPS ambiguity resolution algorithm. Then these double-differenced ambiguity values are introduced as known parameters into the single-differenced model by subtracting from Equation (4). Therefore, the unknown parameters remaining in the GPS observations are the relative receiver clock bias, the integer ambiguity of the base satellite and the errors. From Equation (4), the single-differenced model can be written as Equation (5), assuming that there are four satellites (*j*, *k*, *l*, *m*) available at epoch *t* and satellite *k* is chosen as the base satellite in double-differenced model:

$$\begin{aligned}
\Phi^{j}_{AB}(t) &= \frac{1}{l}\, r^{j}_{AB}(t) + N^{k}_{AB} - f\, d_{AB}(t) + e^{j}_{AB}(t)\\
\Phi^{k}_{AB}(t) &= \frac{1}{l}\, r^{k}_{AB}(t) + N^{k}_{AB} - f\, d_{AB}(t) + e^{k}_{AB}(t)\\
\Phi^{l}_{AB}(t) &= \frac{1}{l}\, r^{l}_{AB}(t) + N^{k}_{AB} - f\, d_{AB}(t) + e^{l}_{AB}(t)\\
\Phi^{m}_{AB}(t) &= \frac{1}{l}\, r^{m}_{AB}(t) + N^{k}_{AB} - f\, d_{AB}(t) + e^{m}_{AB}(t)
\end{aligned} \tag{5}$$

A 'reverse engineering' process is applied to this model in order to produce a reliable estimate of true errors for each satellite. Barnes et al. (1998) and Satirapod (1999) demonstrated the use of this process with the double-differenced model. If the epoch-by-epoch solution is computed, the relative receiver clock bias and the single-differenced integer ambiguity of the base satellite can be eliminated from Equation (5) by subtracting the mean value from the residuals. Hence, the single-differenced error (*e*) for each satellite can be derived and used for a comparison of the two quality indicators.

3. TEST RESULTS AND ANALYSIS

The following series of tests were carried out using data collected on the Mather Pillar on top of the Geography and Surveying building at The University of New South Wales, Sydney, Australia. This station is a GPS permanent station, which has a good observing environment. Test 1 was carried out to investigate the characteristics of SNR. The relationship between SNR and satellite elevation angle information is discussed in the context of Test 2, while a comparative analysis of the two quality indicators is presented in the discussion of Test 3.

3.1 Test 1 -- SNR Characteristics

A zero baseline test was used for this investigation since it was necessary to eliminate any uncertainty due to the use of different antenna types. Three types of receivers were used in the experiment: the Canadian Marconi Corporation Allstar (CMC), the Leica CRS1000 and the NovAtel Millennium. In order to investigate the SNR characteristics for the same receiver type, data was collected by connecting each pair of receivers (of the same type) to the same antenna. Data was collected in static mode for three hours, for each pair of receivers, at a 5-second data rate. C/No values have been presented for the case of two satellites only as the results for the other satellites displayed similar trends. These results have been presented in Figures 1 to 3, which show the time series of the differenced C/No values obtained for the CMC, CRS1000 and NovAtel receivers, respectively.

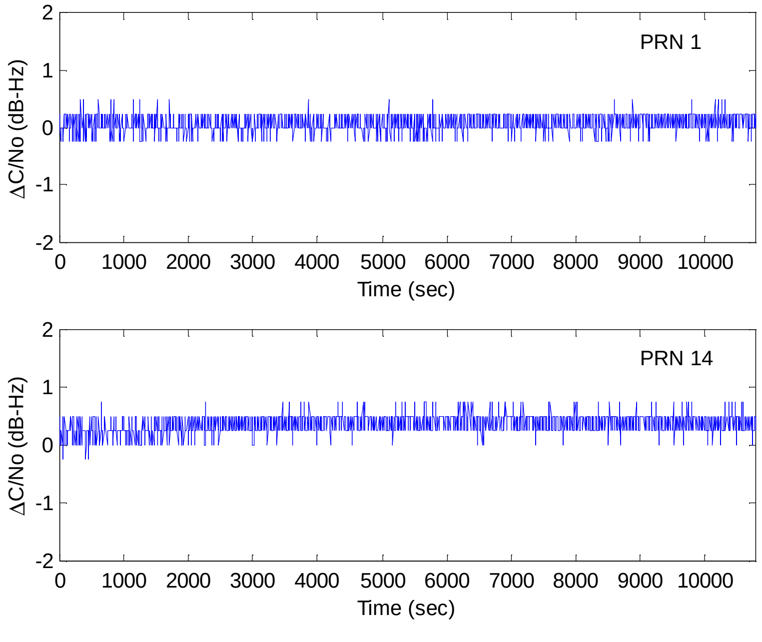

Figure 1 Top: DC/No values between two CMC receivers for PRN 1
Bottom: DC/No values between two CMC receivers for PRN 14

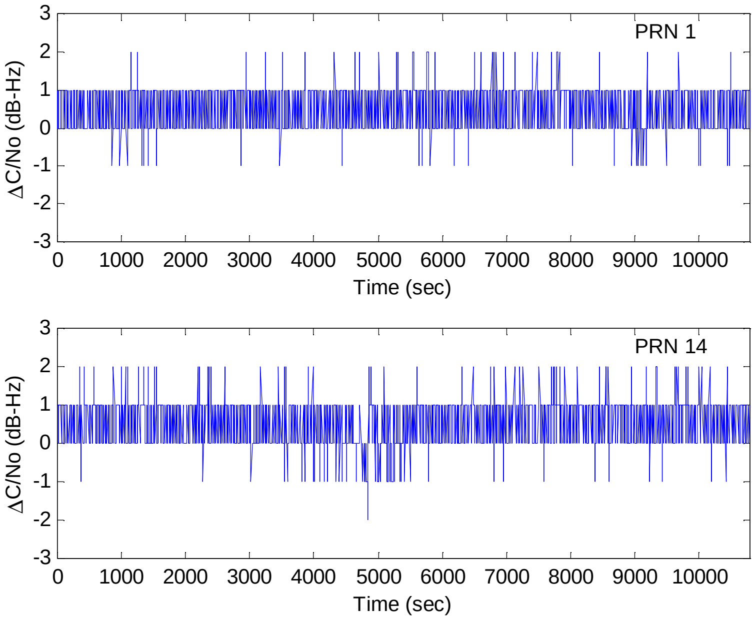

Figure 2 Top: DC/No values between two CRS1000 receivers for PRN 1
Bottom: DC/No values between two CRS1000 receivers for PRN 14

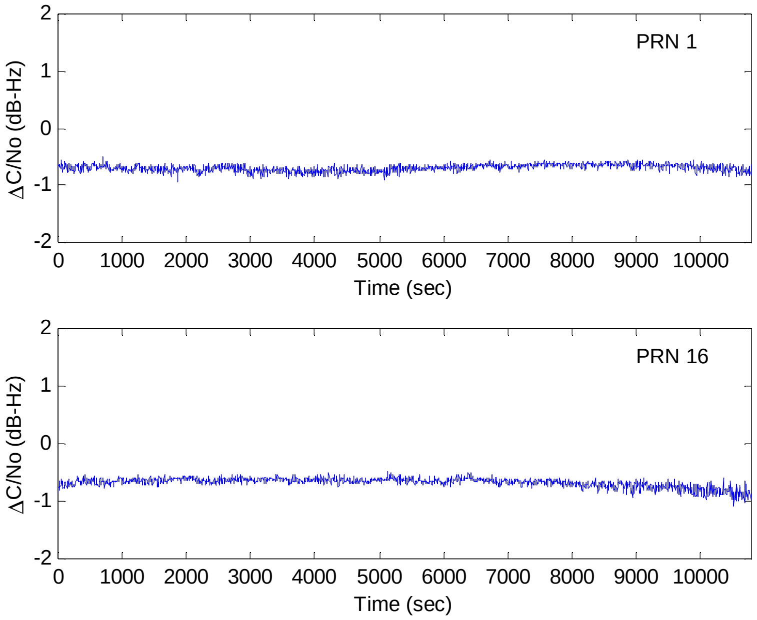

Figure 3 Top: DC/No values between two NovAtel receivers for PRN 1
Bottom: DC/No values between two NovAtel receivers for PRN 16

With reference to Figures 1 to 3, it is clearly evident that, although the same receiver type was used, there is a non-zero difference in C/No values indicating that receivers of the same type nevertheless have different outputs of C/No.

Next, an investigation of the SNR characteristics of the different receiver types was carried out by connecting three types of receivers to the same antenna. The same cable type was also used in this session. Data was collected in static mode for 3 hours at a 1Hz rate. However, the data was sampled every 5 seconds and the results obtained from 3 satellites are presented for comparison. The C/No values obtained from the three receivers are plotted in Figure 4.

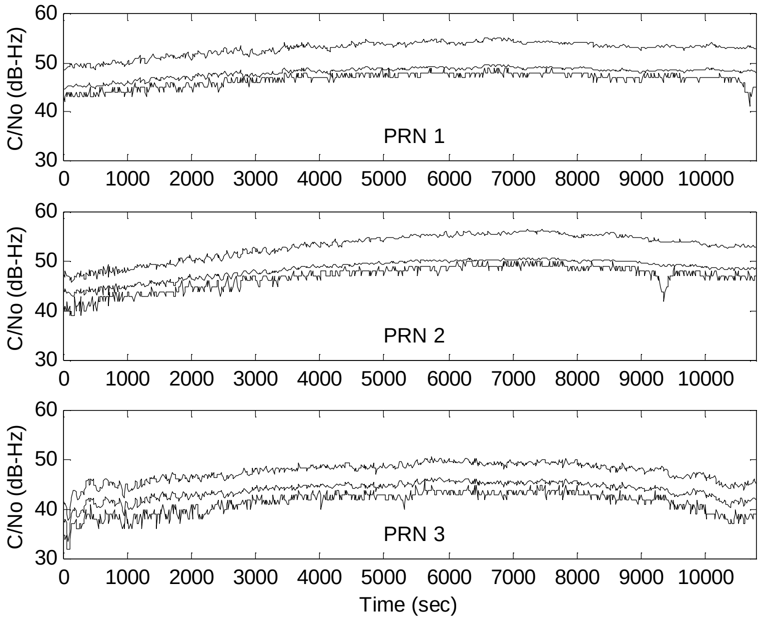

Figure 4 C/No values obtained from three receivers
Top: the C/No values for PRN 1. Middle: the C/No values for PRN 2
Bottom: the C/No values for PRN 3

Figure 4 shows the time series of the C/No values obtained from the three different receivers (CMC, CRS1000 and NovAtel), and the results indicate that there is a difference in C/No values when the different receiver types were used. However, the C/No values show a similar trend for all receivers. Sudden drops in the C/No values are clearly noticeable for satellites PRN1 and PRN2 at different times for the case of the CRS1000 receiver. In order to further investigate this phenomenon it was decided to collect some zero baseline data with the CRS1000 receivers at a 1Hz rate for 2 hours. Selected results for PRNs 3, 29 and 31 are presented in Figure 5.

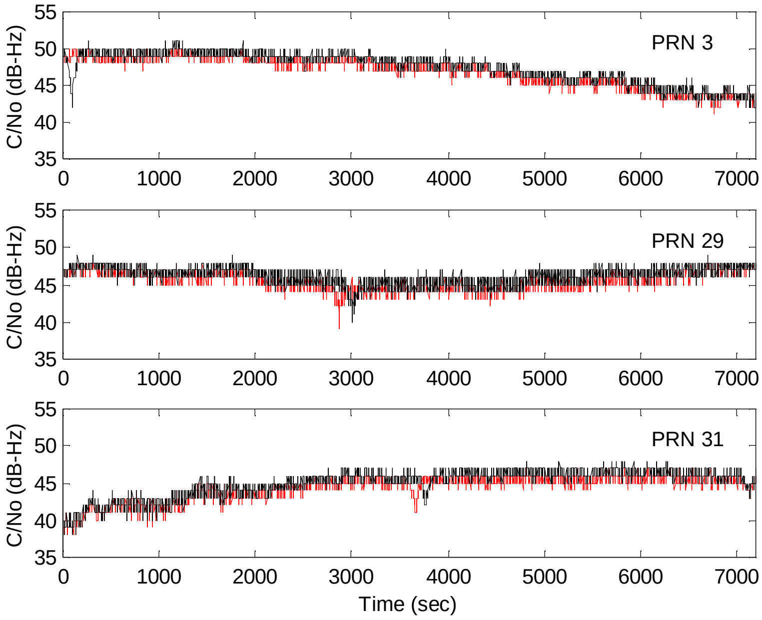

Figure 5 C/No values obtained from CRS1000 receivers
Top: the C/No values for PRN 1. Middle: the C/No values for PRN 29
Bottom: the C/No values for PRN 31(Two subplots for each of the three plots indicate the C/No values for each of the two CRS1000 receivers)

From Figure 5 it appears that these sudden drops are caused by a firmware problem in an individual receiver as they occur at different times, even in the case of the same receiver types. In further analyses, Figures 6(a) and 6(b) show the C/No values against the true errors obtained for the single-differenced model for the case of PRN 3. It is clearly seen that this sudden drop does not reflect any change in true errors (note different C/No values for the two receivers in Figure 6(a)).

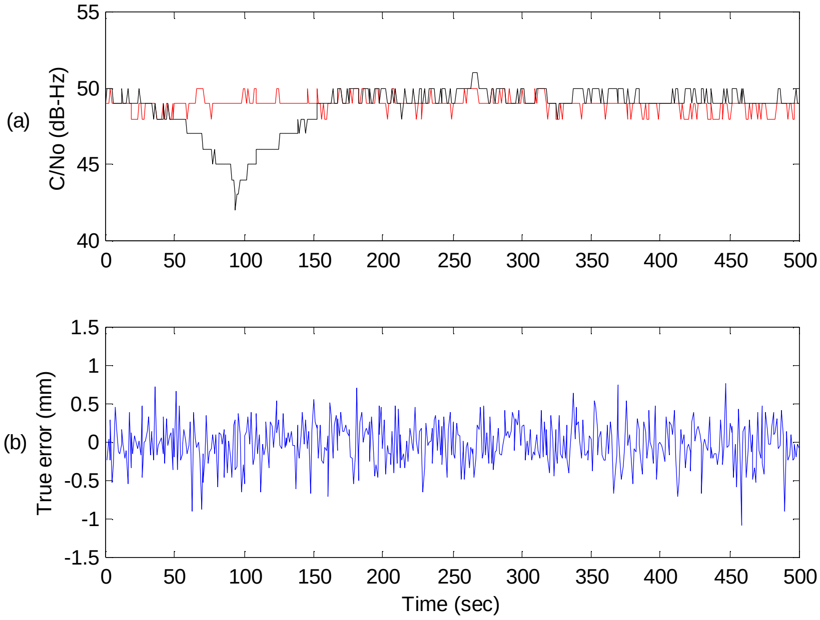

Figure 6 Time series of the C/No values and true errors obtained from CRS1000 receivers (a) The C/No values for PRN 3 (b) The true errors for PRN 3

3.2 Test 2 -- SNR & Satellite Elevation

The relationship between SNR and satellite elevation angle data has been mentioned by several authors (for example, Hartinger & Brunner, 1998; Lau & Mok, 1999), and it is generally accepted that SNR is almost directly proportional to satellite elevation. However, it is necessary to examine this relationship more closely before advocating the use of these quality indicators. An experiment was therefore set up from which several zero baseline data sets were collected in static mode for over 16 hours. The data set obtained from the CMC receiver was selected for the analysis. From this data set, four satellites were selected to study this relationship. The TEQC software was then used to check for the presence of any multipath disturbance, and it was found that there is no significant multipath effect on the satellite signals. The time series of the C/No values and the satellite elevation angle data are plotted in Figures 7 and 8. Figure 7 shows that the relationship between SNR and satellite elevation is indeed as established by previous studies. Figure 8, however, shows that this relationship may not be true for high satellite elevations.

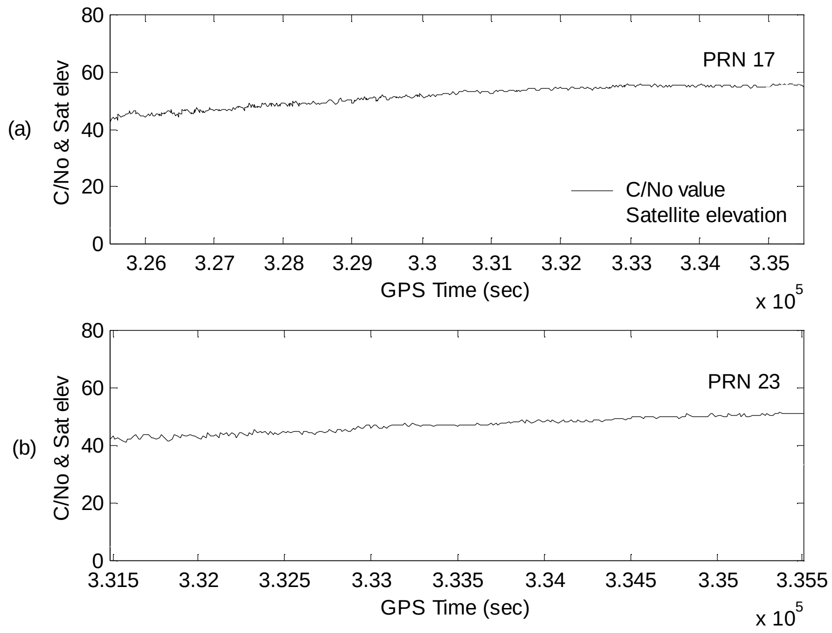

Figure 7 Time series of the C/No values and satellite elevation data obtained from the CMC receiver
(a) The C/No values and satellite elevation data for PRN 17
(b) The C/No values and satellite elevation data for PRN 23

It is also evident that the C/No value for low satellite elevation in some instances is higher than the C/No value at high satellite elevation (see the two peaks in Figure 8(a) and 8(b)). Thus, the two standard quality indicators do not always follow the same trend. Further comparative analysis of these two quality indicators therefore was necessary, and the results are presented in the next section.

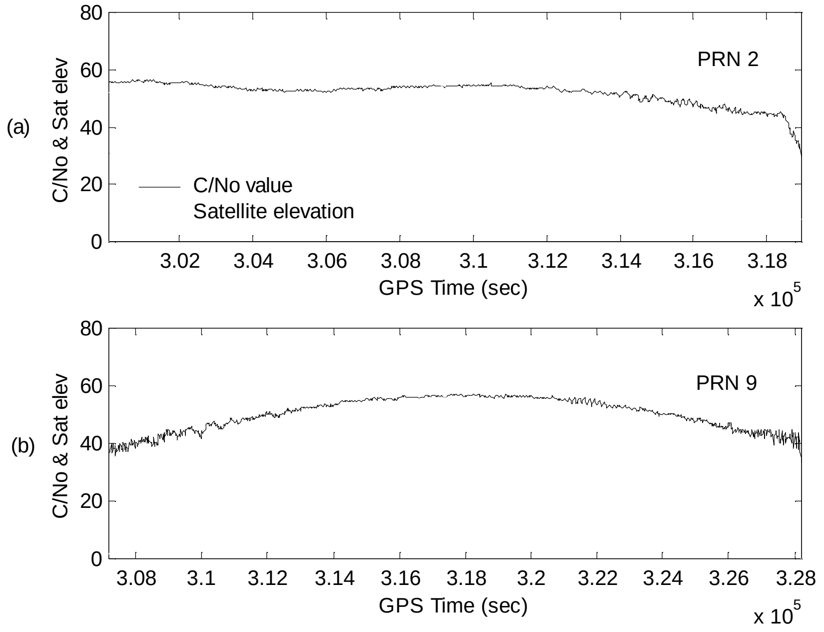

Figure 8 Time series of the C/No values and satellite elevation data obtained from CMC receiver
(a) The C/No values and satellite elevation data for PRN 2
(b) The C/No values and satellite elevation data for PRN 9

3.3 Test 3 -- The Comparative Analysis

The aim of this experiment was to compare the two quality indicators with the known true errors through the use of single-differenced observables. Single-differenced observables were chosen as both the SNR and satellite elevation angle data are 'one-way observations'. Therefore, the validity of these quality indicators should be assessed on a satellite-by-satellite basis. A previous investigation on the relationship between phase noise and satellite elevation angles based on double-differenced residuals was reported in Cannon (1998). For this experiment, zero baseline data was collected for a variety of receiver types. Each of the data sets was divided into 0.5-hour sessions and then processed using the single-differenced model described in section 2.3. The standard deviation of the true errors and the mean values of C/No and satellite elevation are the quantities of interest in this analysis. The results from the different receivers show similar trends, and selected results from the experiments are plotted in Figures 9 and 10.

Figures 9 and 10 show the relationship between the two quality indicators and the standard deviation of true errors for the CRS1000 and NovAtel receivers, respectively. For each of the figures the top chart shows the relationship between the mean C/No values and the standard deviation of the true errors, while the bottom chart shows the relationship between the mean satellite elevation values and the standard deviation of the true errors.

From Figures 9 and 10 it can be seen that the C/No values reflect a more realistic trend than those based on satellite elevation data, which show a larger discrepancy. However, it can also be seen that in some cases both of these indicators fail to reflect reality, as they do not match the standard deviation of true errors. Moreover, SNR and satellite elevation data are dependent upon the receiver type, therefore in order to use these data efficiently further investigations will be necessary.

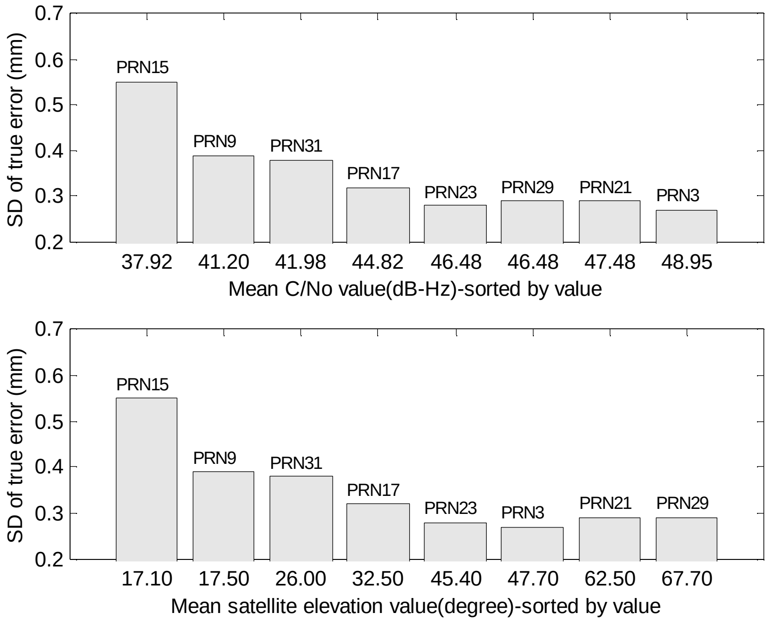

Figure 9 Comparison of the two quality indicators for CRS1000 receivers (zero baseline)
Top: Mean C/No values and standard deviation of true errors
Bottom: Mean satellite elevation values and standard deviation of true errors

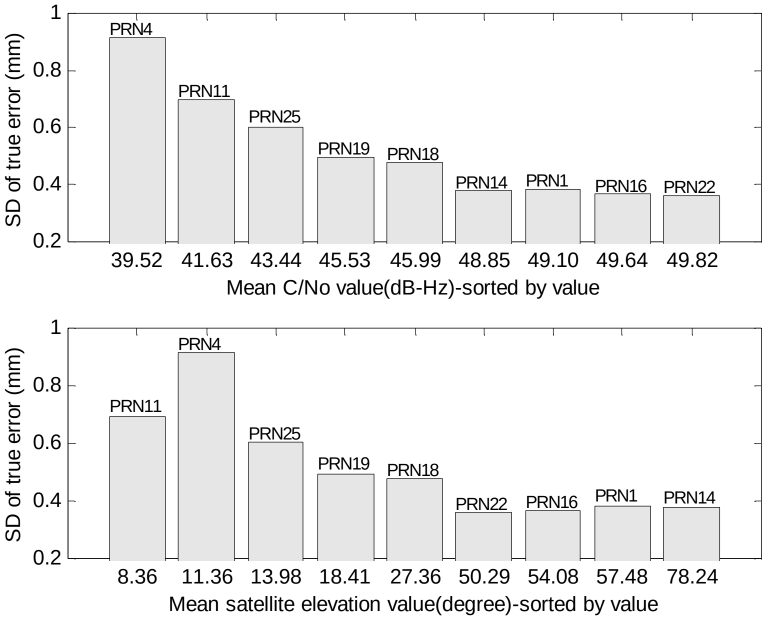

Figure 10 Comparison of the two quality indicators for NovAtel receivers (zero baseline)
Top: Mean C/No values and standard deviation of true errors
Bottom: Mean satellite elevation values and standard deviation of true errors

4. ARE THESE INDICATORS REALLY USEFUL?

In rapid static and kinematic GPS positioning, the issue of the stochastic model becomes critical. A realistic stochastic model will lead to an improvement in the accuracy and reliability of GPS results. In such 'high productivity' GPS techniques many researchers have extensively used the two standard quality indicators to refine the stochastic model, hoping to obtain a more reliable solution. However, from our admittedly limited investigations, both quality indicators did not always reflect reality as far as data quality is concerned. This can be partially attributed to the fact that although the relationships defined by Equations (2) and (3) are empirically derived from extensive data sets, they are still largely dependent on the data originally used for their derivation. In Wang et al. (1998), a more rigorous statistical method called MINQUE (Minimum Norm Quadratic Unbiased Estimation -- Rao, 1971) is used to construct a more realistic stochastic model, although the physical correlations are still ignored. As discussed by many researchers (for example, Barnes et al.; 1998; El-Rabbany, 1994; Wang, 1998), the two types of physical correlations in GPS observations, temporal correlation and spatial correlation, can be used to significantly improve the accuracy and reliability of GPS results. Therefore, in order to further improve the stochastic model it is necessary to incorporate these correlations into the generation of the variance-covariance (VCV) matrix of the observations. Consequently, the construction of a realistic stochastic model becomes more complex and remains one of the biggest challenges for GPS research. Future work will attempt to define a rigorous technique for generating realistic stochastic models for GPS processing.

5. CONCLUDING REMARKS

It is necessary that we develop a better understanding of quality indicators for GPS observations. In this paper, the standard quality indicators for GPS observations, SNR and satellite elevation angle information have been investigated. A comparative analysis was carried out in an attempt to verify the previously established relations between these indicators. Based on the results obtained, the following comments can be made:

- There is a difference in the C/No values for the same receiver types, as well as for different receiver types.
- Sudden drops occurred in the C/No values for a specific receiver type in the case of high elevation satellites. These sudden drops may result in a misrepresentation of the quality of the measurements if the C/No values are used as a quality indicator.
- The two standard quality indicators do not always follow the same trend.
- C/No values reflect a more realistic trend than satellite elevation angle data.
- In general, both C/No values and satellite elevation angle data can be used as quality indicators, but they do not always reflect reality. More rigorous quality indicators therefore need to be developed.

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