Interpolation of GPS Results
Incorporating Geophysical and InSAR Information

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ABSTRACT

Continuous GPS networks, typically with a station spacing of about 30km, are still not dense enough to accurately characteristise the dynamics of active faults. Interpolation of these GPS results can improve our understanding of active faults and hence promote related studies. Moreover, even when the networks are densified in order to recover the signature of active faults, the station configuration design may not be ideal. Interpolation at these points, based on the GPS results from a well-designed station network, can provide a good quality control measure.

As a first step in the interpolation process an irregular grid pattern is formed, based on the locations of the GPS stations, by using the indexed sorting algorithm. In order to interpolate objectively, the GPS stations and the intended interpolating points are classified into different sub-regions according to their positions in relation to the faults, which are expressed by open- and closed-curve models. GPS results from stations in the same sub-region are used to derive a dynamic model for interpolation at grid points in the same sub-region. A deformation distribution model based on GPS and differential Synthetic Aperture Radar Interferometry (InSAR) results is used as constraints to scale the time series generated using the dynamic model.

1. INTRODUCTION

Large-scale continuous GPS (CGPS) networks for geodynamics have been established over the past decade. However, these networks, the densest of which typically have station spacings of about 30km, are still not dense enough to accurately characteristise the dynamics of active faults. In fact, if one inspects the resolution requirements for some geophysical and geological applications (as shown in Fig. 1, where the coverage of the current CGPS is indicated by a dashed-line rectangle), the majority of the applications remain unsatisfied. If the rectangle is extended in the negative direction of the vertical axis, this represents a temporal densification of the GPS measurements [Ge et al., 1999]. If the rectangle is extended in the negative direction of the horizontal axis, this is a spatial densification of the GPS measurements. This paper will focus on "soft" densification, i.e. spatial densification without deploying more GPS hardware. "Hard" densification, i.e. spatial densification by deploying more GPS receivers (e.g., low-cost, single-frequency receivers) will be discussed in a separate paper [Rizos et al., 1999]. Both temporal and spatial densification of GPS measurements is necessary in order to address all the applications.

Interpolation of GPS results (spatial densification especially) can improve our understanding of active faults and hence promote related studies associated with earthquake modelling and hazard mitigation. Moreover, even when the networks are densified in order to recover the signature of active faults, the station configuration design may not be ideal. Interpolation at these points, based on the GPS results from a well-designed station network, can provide a good quality control measure.

In order to interpolate objectively both a dynamic model, incorporating geophysical information such as active faults, and a distribution model, incorporating Interferometric Synthetic Aperture Radar (InSAR) information, have to be developed.

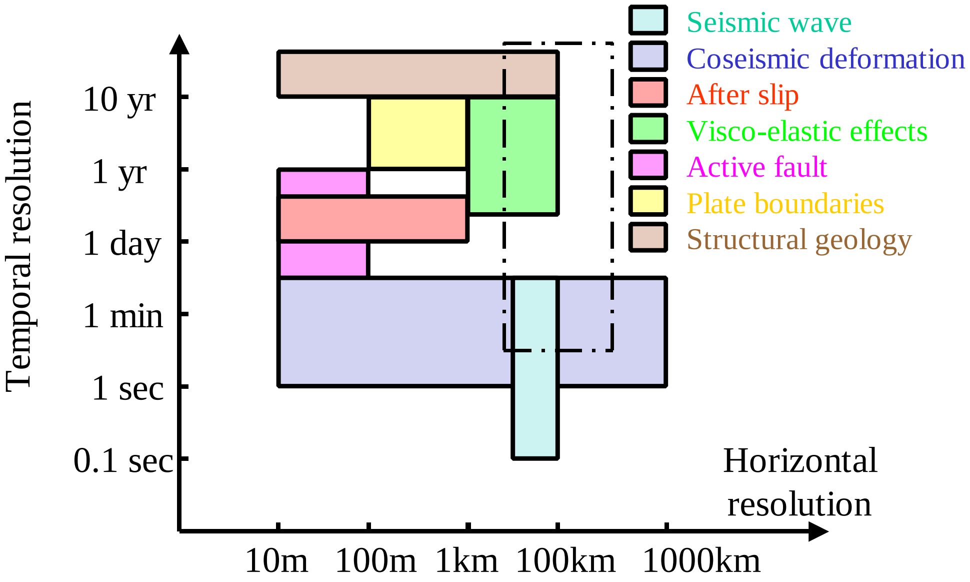
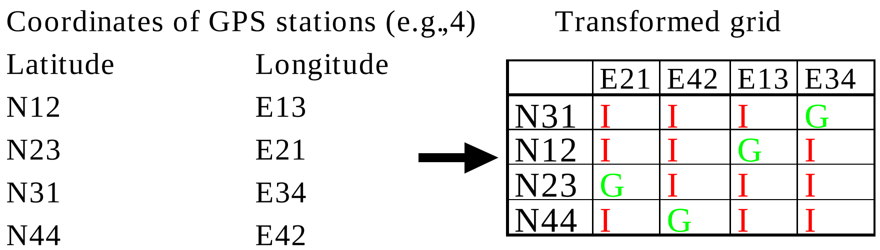


Figure 1. Resolution requirements of some geophysical and geological applications (various sources).

2. DYNAMIC MODEL FOR INTERPOLATION INCORPORATING GEOPHYSICAL INFORMATION

As a first step in the interpolation process, an irregular grid pattern is formed based on the locations of the stations in the GPS network. As shown in Fig. 2, the latitudes and longitudes of the GPS stations are read from a data file. Then the latitudes and longitudes are sorted according to their values. The sorted latitudes and longitudes are then combined to form the grid, consisting of both existing GPS stations (denoted by “G”) and points to be interpolated (denoted by “I”). The important feature of the two dimensional sorting is that the attributes of GPS stations are maintained so that no interpolation operation is needed on grid points of GPS stations. Therefore, the algorithm carries out “indexed sorting”.



Coordinates of GPS stations (e.g.,4)

Latitude	Longitude
N12	E13
N23	E21
N31	E34
N44	E42

Transformed grid

	E21	E42	E13	E34
N31	I	I	I	G
N12	I	I	G	I
N23	G	I	I	I
N44	I	G	I	I

Figure 2. An irregular grid formed by indexed sorting.

In order to interpolate objectively, it is proposed that geophysical information then be incorporated. It is well known that the movements of GPS stations on the two sides of a fault can be significantly different therefore it is important to classify the stations and the intended interpolating points according to their positions in relation to the fault. The open- (Fig. 3) and closed- (Fig. 4) curve models have been designed to describe the faults. In the open-curve model as shown in Fig. 3, the region of interest is divided into upper (or left) and lower (or right) sub-regions by an open curve. This model should be adequate for most seismic faults.

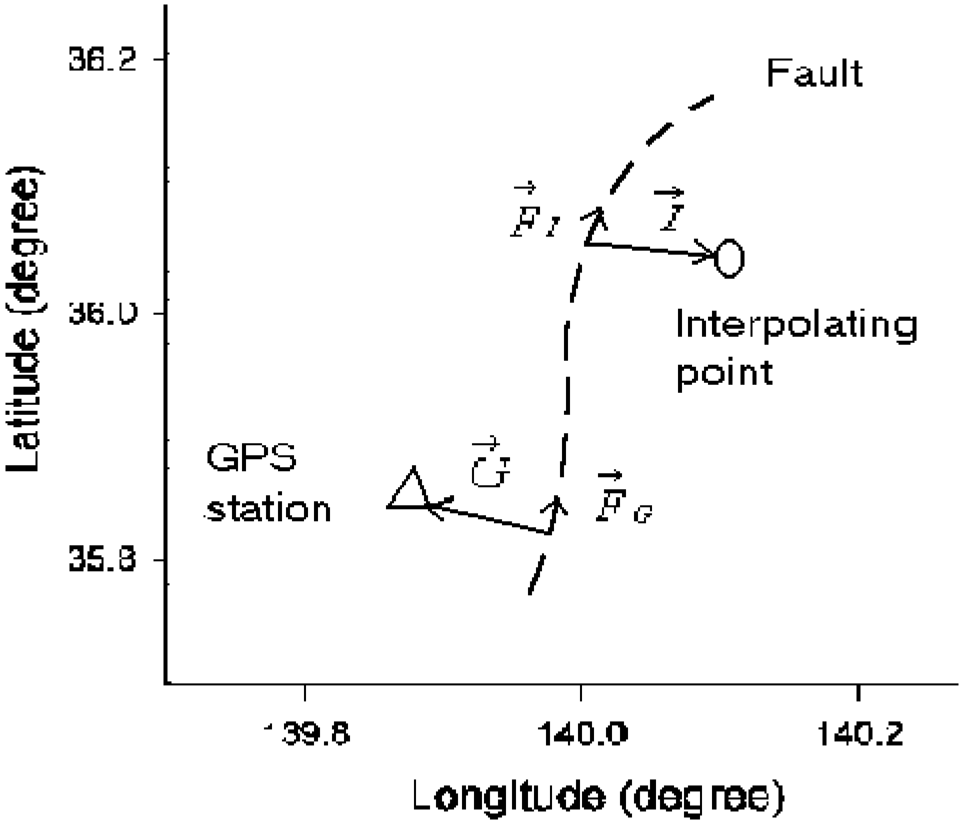


Figure 3. Open-curve model: one GPS station and one interpolating point case.

An algorithm has been developed to automatically identify in which sub-region a station or an interpolating point belongs to. Assume the unit vectors of latitude is $\vec{i}$ and longitude is $\vec{j}$, from an arbitrary point on the fault (N_i, E_i) to the GPS station (GS) at (N_G, E_G) the vector can be expressed as:

$$\vec{G} = (N_G - N_i) \bullet \vec{i} + (E_G - E_i) \bullet \vec{j} \qquad (1)$$

A minimum search of vector length is performed to find $|\vec{G}|_{(N_i, E_i)} = \min$

Therefore, a GS local fault vector can be written as:

$$\vec{F}_G = (N_{i+1} - N_i) \bullet \vec{i} + (E_{i+1} - E_i) \bullet \vec{j} \qquad (2)$$

From an arbitrary point on the fault (N_j, E_j) to the interpolating point (IP) at (N_I, E_I) the vector can be expressed as:

$$\vec{I} = (N_I - N_j) \bullet \vec{i} + (E_I - E_j) \bullet \vec{j} \qquad (3)$$

A minimum search of vector length is performed to find $|\vec{I}|_{(N_j, E_j)} = \min$

Then, an IP local fault vector can be written as:

$$\vec{F}_I = (N_{j+1} - N_j) \bullet \vec{i} + (E_{j+1} - E_j) \bullet \vec{j} \qquad (4)$$

From Equations (1) to (4) a “decision” can be made: if $(\vec{F}_G \times \vec{G}) \bullet (\vec{F}_I \times \vec{I})$ >0 the GPS station and the interpolating point are on the same side of the fault. Otherwise they are on different sides of the fault.

In the closed-curve model as shown in Fig. 4, the region to be studied is divided into outside and inside sub-regions by a closed curve. This model is appropriate for applications such as volcano deformation monitoring.

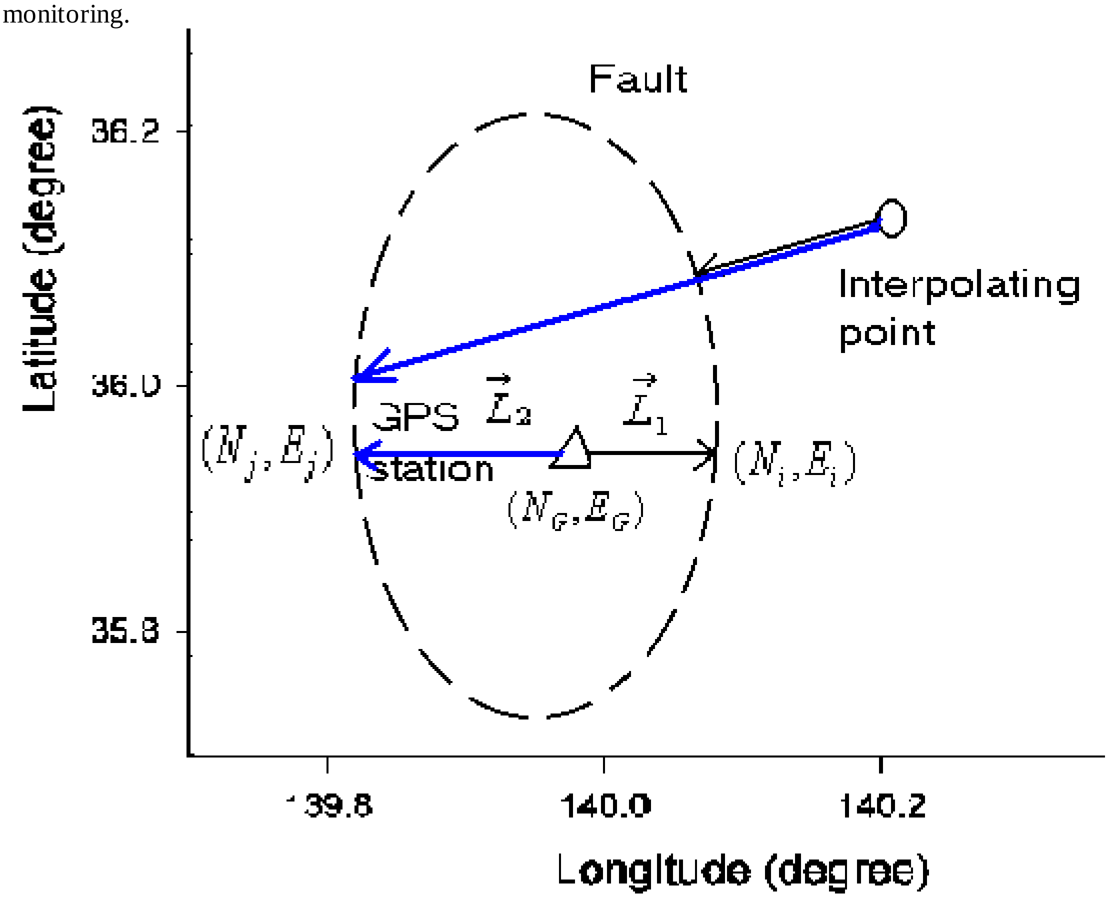


Figure 4. Closed-curve model: one GPS station and one interpolating point case.

A different algorithm for this model is developed to automatically identify in which sub-region a station or interpolating point belongs to. Again assume the unit vectors of latitude is $\vec{i}$ and longitude is $\vec{j}$. From the GPS station (GS) at (N_G, E_G) to an arbitrary point on the fault (N_i, E_i) the vector can be expressed as:

$$\vec{G} = (N_i - N_G)\bullet \vec{i} + (E_i - E_G)\bullet \vec{j}$$

A minimum search of vector length is performed to find $|\vec{G}|_{(N_i,E_i)} = \min$

The GS primary vector is thus determined as: $\vec{L}_1 = \vec{G}$ (5)

Then a minimum distance search from an arbitrary point on the fault to the GS primary vector can determine (N_j, E_j) which is used to calculate the GS secondary vector:

$$\vec{L}_2 = (N_j - N_G)\bullet \vec{i} + (E_j - E_G)\bullet \vec{j} \qquad (6)$$

From Equations (5) and (6), a "decision" can be made: if $\vec{L}_1 \bullet \vec{L}_2 > 0$ the GPS station is outside of the fault. Otherwise it is inside the fault. The position of the interpolating point relative to the fault can be determined in the same way.

The combination of open- and closed-curve models can deal with comparatively complex fault systems. After the classification, GPS results from stations in the same sub-region are used to derive a dynamic model, which is used to interpolate at grid points in the same sub-region. Fig. 5 is an example of a dynamic model extracted using adaptive filtering [Ge, 1999] on CGPS results from two closely located stations in the same classified group: stations BRAN and LEEP of the SCIGN.

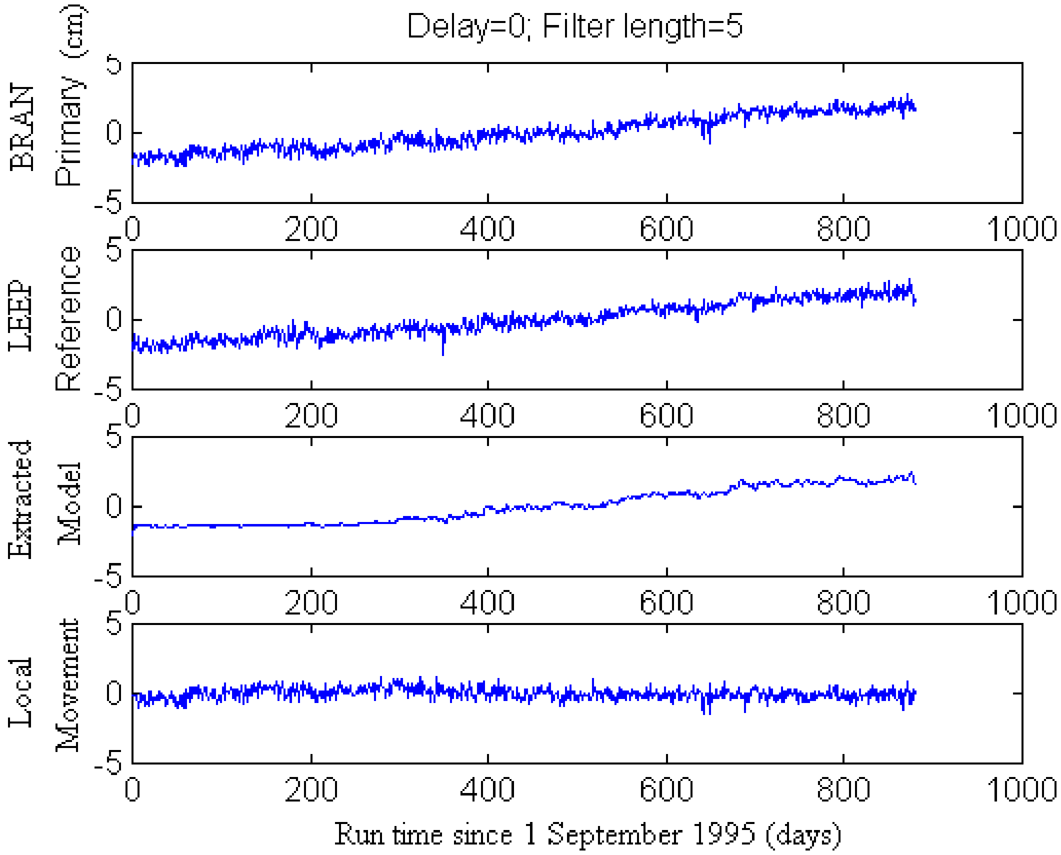


Figure 5. Dynamic model extracted using adaptive filtering on CGPS results from BRAN and LEEP stations of SCIGN (latitude component, data courtesy of JPL).

3. DISTRIBUTION MODEL FOR INTERPOLATION INCORPORATING INSAR INFORMATION

Interferometric Synthetic Aperture Radar (InSAR) is a technique first suggested in 1974 [Graham, 1974]. After developments over more than two decades, InSAR is now capable of addressing many applications such as mapping topography, detecting topographic change, etc. InSAR has approximately 25m spatial resolution. Without the need for any ground-based receiver or cooperative target, it can virtually monitor every corner of the earth. But InSAR is very sensitive to errors such as due to atmospheric effects (tropospheric delay, ionospheric delay, etc.), satellite orbit error, conditions of the ground surface and temporal decorrelation. When presented in the InSAR image, these errors can lead to misleading interpretation. There is at present no way to eliminate them using SAR data alone.

Therefore, a distribution model for interpolation based on both GPS and InSAR results is proposed, as illustrated in Table 1. In some sites of interest there are both GPS and InSAR results for the deformation. But in most other sites there will only be InSAR-derived results. All of the results are input into a least-square adjustment. The adjusted results are used as constraints to scale the time series generated at the interpolated points based on the dynamic model. It is important to have overlapping GPS and InSAR results over the active fault region so that the GPS results can be used to calibrate out some errors in the InSAR results. But this condition is very hard to satisfy [Bock and Williams, 1997]. It is hoped that the densification of CGPS and new missions for InSAR such as the Shuttle Radar Topography Mission (SRTM), Airborne Synthetic Aperture Radar (AIRSAR), Geographic Synthetic Aperture Radar mission (GeoSAR), and Lightweight Synthetic Aperture Radar (LightSAR) [JPL, 1999], will ease such difficulties.

Table 1. Distribution model incorporating InSAR information

Interested Site	GPS Result	InSAR result	Least-square adjusted result
1	G1	S1	A1
2	G2	S2	A2
3	G3	S3	A3
4	G4	S4	A4
5		S5	A5
6		S6	A6

4. CONCLUDING REMARKS

Geophysical and InSAR information have been incorporated in the interpolation of GPS measurements. The indexed sorting algorithm is very effective in forming an irregular grid pattern for interpolation and maintaining the “attributes” of the GPS stations. The combination of open- and closed-curve models of a fault is suitable for dealing with comparatively complex fault systems. Adaptive filtering has been successfully used to extract a dynamic model from the GPS time series of stations in the same classified group. However, much more effort is needed to construct an accurate distribution model due to the lack of overlapping GPS and InSAR-derived results over the same active fault zone.

The work described in this paper is part of a larger study on the use of the Double Interpolation and Double Prediction (DIDP) scheme to integrate GPS and InSAR results.

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REFERENCES

Bock, Y., and S. Williams, 1997. Integrated satellite interferometry in Southern California. EOS Trans., AGU, Vol.78, No.29, 293.

Ge, L., S. Han, C. Rizos, Y. Ishikawa, M. Hoshiba, Y. Yoshida, M. Izawa, N. Hashimoto, and S. Himori, 1999. GPS seismometers with up to 20Hz sampling rate. Pres. *International Symposium on GPS - Application to Earth Sciences and Interaction with Other Space Geodetic Techniques*, Tsukuba, Japan, 18-22 October.

Ge, L., 1999. GPS seismometer and its signal extraction. Proc. *12th Int. Tech. Meeting of the Satellite Division of the U.S. Inst. of Navigation GPS ION'99*, Nashville, Tennessee, 14-17 September.

Graham, L.C., 1974. Synthetic interferometer radar for topographic mapping. Proc. IEEE, 62, 763-768.

JPL, 1999. http://www.jpl.nasa.gov/missions/future/.

Rizos, C., S. Han, L. Ge, H.Y. Chen, Y. Hatanaka, and K. Abe, 1999. Low-cost densification of permanent GPS networks for natural hazard mitigation: first tests on GSI's GEONET network. Pres.

International Symposium on GPS - Application to Earth Sciences and Interaction with Other Space Geodetic Techniques, Tsukuba, Japan, 18-22 October.