Slope Stability Monitoring
By The Use Of GPS Multipath As A Signal

Linlin GE and Chris RIZOS

School of Geomatic Engineering
The University of New South Wales
Sydney NSW 2052, AUSTRALIA
Phone: +61-2-9385 4177
Email: l.ge@unsw.edu.au

Abstract. GPS multipath has long been considered a major error source in GPS applications. In this paper, however, an attempt is made to use multipath as a *signal*. The proposed application of using GPS multipath to monitor the slope stability involves two steps. The first step is to extract multipath and to detect its change based on an adaptive filter using the least-mean-square algorithm. In the second step, the mathematical models for slope stability monitoring using the change of GPS multipath effects have been established. In addition to the monitoring of slope stability, this technique provides an easy-to-implement quality assurance tool for CGPS antenna environment sensing after disasters such as typhoons, cyclones, and earthquakes.

Keywords. slope stability, GPS multipath, adaptive filter.

1 Introduction

No matter how well continuously operating GPS (CGPS) networks are designed, multipath is a significant concern as it impacts on the quality of the CGPS outputs or 'products'. Moreover, it may change slowly on a seasonal basis, or abruptly due to events such as a snowfall.

Recently effort has also been made to make use of multipath, i.e. to use it as a signal rather than a noise (e.g. Ding et al., 1999). The idea of analysing GPS errors and biases for signal is not a new one. The best example is the so-called "GPS Meteorology", which uses the atmospheric refraction effects on GPS measurements as its input. Therefore, instead of focusing on multipath mitigation, in this paper GPS multipath is used as a signal to sense the change in the GPS antenna environment. This capability is very useful in the maintenance and quality assurance of CGPS networks such as the GEONET of Japan, where the GPS monitoring environment can be changed not only by natural phenomenon such as snow accummulation and tree growth, but also by disasters such as slope failure, volcanic eruption, typhoon and earthquake.

2 Multipath Change Detection Procedures

As illustrated in Fig. 1, there are three main steps in implementing multipath change detection based on adaptive filtering (Ge et al, 2000). The first step is to derive the pseudo-range or carrier phase multipath residual time series through multipath combination of the GPS measurements. In the second step, the adaptive filter (indicated by the star in Fig. 1) is used first to extract the multipath from the residual series for two consecutive days, obtained from the multipath combination. Two multipath series will be obtained for each 3 day set as a result. In the third step, the adaptive filter is used again to detect the multipath change between the two multipath series derived in the second step.

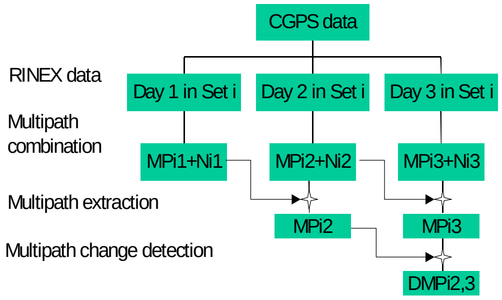

Figure 1. Multipath change detection procedures.

In the following two sections this technique is first used for multipath change detection in a stationary antenna environment to test if the detected change is zero when there is no change in the antenna environment. Then it is applied for multipath change detection in a changed antenna environment to test if the technique can identify the change when there is indeed a change in the multipath disturbance caused by snowfall.

3 Multipath Change Detection in a Stationary Antenna Environment

Two data sets were used in this test, which were the same as those used in the Han & Rizos (2000) study. In the first data set, the pseudo-range data over a period of nearly 3 hours for four successive days, collected on the roof of the Geography and Surveying (GAS) building, at the University of New South Wales (UNSW), from 30 September to 3 October 1997, using an Ashtech Z12 GPS receiver, were used to compute the pseudo-range multipath time series. (In this study only the 'pseudo-range multipath time series' for L1 for satellite PRN 9 are used.)

In the second data set, multipath series of double-differenced carrier phase measurements over a period of nearly 2 hours on four successive days for two satellites (PRNs 1 and 21) were calculated from data collected in an experiment carried out on the roof of the GAS building at UNSW, from 28 April to 1 May 1997, using two Leica SR299 GPS receivers with a baseline length of about 6 metres. Here the residual series of double-differenced carrier phase observations are used as the 'carrier phase multipath time series' because they reflect the multipath disturbance and observation noise (it is assumed that all the other errors and biases are negligible because the baseline length is only of the order of 6 metres).

Table 5-1 summarizes the standard deviations (STD) of the pseudo-range multipath extraction (the first three rows) and change detection (the last two rows) results. Note that in the multipath extraction step, the incoherent output is the uncorrelated noise and the coherent output is multipath, while in the multipath change detection step the incoherent output is the change of multipath and the coherent output is correlated multipath. Note that the changes of multipath detected are not zero as they should be, but are 0.044 and 0.046m in the pairs Day 2 & Day 3 and Day 3 & Day 4 respectively, which are nevertheless smaller than the derived uncorrelated noise. Therefore, a tentative conclusion is that any multipath change smaller than the uncorrelated noise (mainly the receiver noise) will go undetected.

Table 2 summarizes the standard deviations (STD) of the carrier phase multipath extraction (the first three rows) and change detection (the last two rows) results. Again note that in the multipath extraction step, the incoherent output is the uncorrelated noise and the coherent output is multipath while in the multipath change detection step the incoherent output is the change of multipath and the coherent output is correlated multipath. Note that the changes of multipath detected are not zero as they should be, but are 0.966 and 1.010mm in the pairs Day 2 & Day 3 and Day 3 & Day 4 respectively, which are nevertheless smaller than the derived uncorrelated noise. Therefore, a tentative conclusion, similar to the pseudo-range case, is that any carrier phase multipath change smaller than the uncorrelated noise (mainly the receiver phase noise) will go undetected.

Therefore, one conclusion is that multipath change (either in pseudo-range or carrier phase) below the receiver noise level (in the case of this study: code: 0.05m; phase: 1.5mm) could not be detected using this technique.

Table 1. Multipath Extraction and Change Detection: Pseudo-Range Result.

Day	Incoherent Output STD (m)	Coherent Output STD (m)
2	0.046	0.210
3	0.047	0.205
4	0.052	0.212
2-3	0.044	0.201
3-4	0.046	0.207

Table 2. Carrier Phase Multipath Extraction and Change Detection Result.

Day	Incoherent Output STD (mm)	Coherent Output STD (mm)
2	1.543	2.367
3	1.409	1.866
4	1.457	1.969
2-3	0.966	1.624
3-4	1.010	1.718

4 Multipath Change Detection in a Changed Antenna Environment

To test the capability of this technique in detecting multipath change, continuous GPS data from two Japanese GEONET stations 960627 and 92110 located in the Tsukuba City were used. In both Stations 960627 and 92110, a Trimble 4000SSI receiver and a Permanent L1/L2 antenna are installed. The distance between the two stations is about 286m.

An abnormal change as large as 3cm in the time series of the baseline length between the two stations was detected, which corresponds to the time of a snowfall, as illustrated in Fig. 2.

In order to test whether the adaptive filtering technique can detect this change in the antenna environment, the CGPS data were processed following the procedures outlined in the previous sections.

Fig. 3 is the pseudo-range multipath STD change on the C1 code at Station 960627 for satellite PRN25. The total change (the biggest one in the figure) in the multipath STD from DOY 64 to 65 is 0.19m, while the second biggest change is 0.10m from DOY 87 to 88, which is an almost 100% increase.

Fig. 4 is the result of carrier phase multipath change on L1 in DD between Stations 92110 and 960627, for satellite pair PRNs 22 and 25. The biggest change occurs on DOY 64 to 65.

Therefore, it seems that when there is indeed a change in the antenna environment it will be detected using this technique.

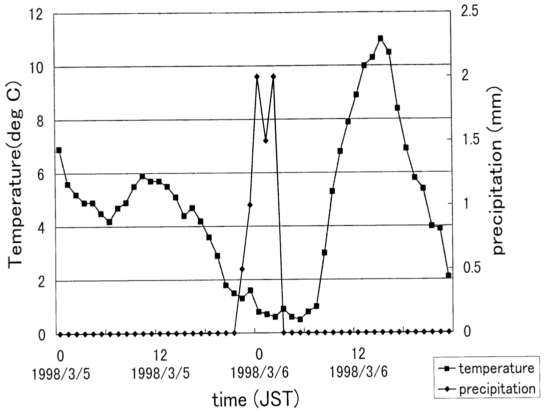

Figure 2. Precipitation and temperature in Tsukuba from 5 March (DOY 64) to 6 March (DOY 65) 1998.

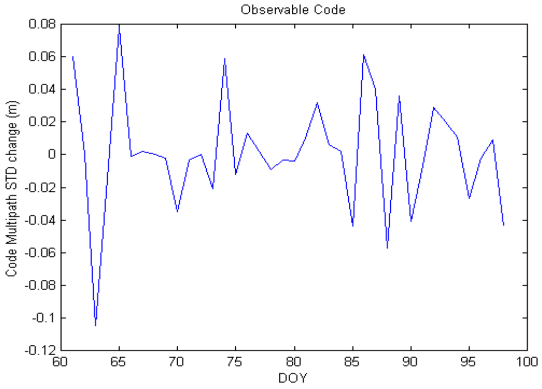

Figure 3. Pseudo-range multipath change on C1 code at Station 960627 for satellite PRN25.

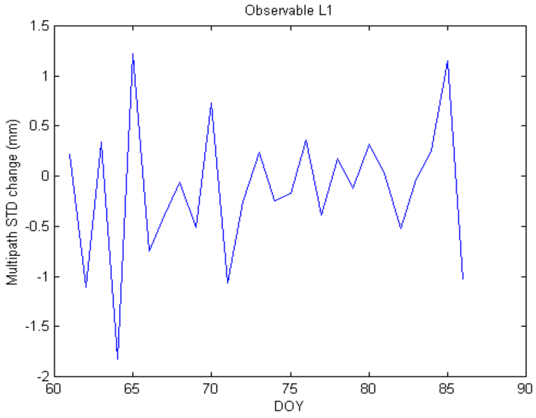

Figure 4. Carrier phase multipath change on L1 in DD between Stations 92110 and 960627 for satellite pair PRNs 22 and 25.

5 Slope Stability Monitoring Using Multipath Change

Traditional slope stability monitoring equipment and techniques include:

- Survey techniques (e.g., EDM and GPS leveling or photogrammetric surveys);
- Displacement monitoring pins and tape extensometers fixed across cracks or major rock defects;
- Borehole inclinometers; and
- Extensometers anchored within the rock mass via boreholes drilled into slopes.

Compared to these techniques, there are obvious advantages in using the multipath effect for slope stability monitoring. Multipath signals have common transmission time at the satellite, almost common course through the atmosphere and are received by the same receiver at the same time. Therefore, they are less noisy than the satellite-receiver double-differenced observable. This ensures that the technique could achieve very high accuracy. Other advantages include the movement of the slope at multiple reflecting points can be monitored, which contributes to higher efficiency; moreover, the technique is relatively cheap and highly automatic because a GPS receiver is a general purpose equipment compared to the equipment used in some of the traditional techniques.

In this Section the mathematical models necessary to employ the GPS multipath effect for slope stability monitoring are given.

5.1 Models for relating multipath to slope movement

Fig. 5 is a slope in the 3D rectangular coordinate system monitored by the GPS multipath. In the figure, the slope to be monitored is represented by a plane containing the y-axis and point *R*. The slope is in angle q to the horizontal plane *xoy*. j is the orientation of the slope. C'C is the direct line-of-sight signal from a GPS satellite and RC is the reflected signal by the slope. The GPS antenna phase centre C is *d* metres away from the foot of the slope and *h* metres above the ground OE.

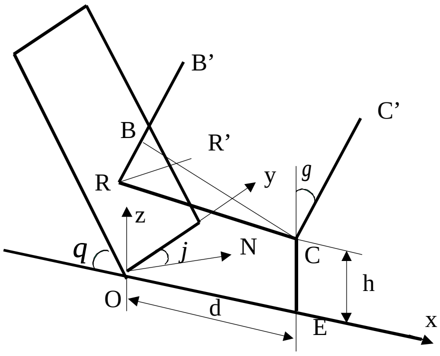

Figure 5. A slope in the 3D rectangular coordinate system.

The geometric path length difference between the direct and reflected signals is (Ge, 2001):

$$L = (1 + \cos 2d) \cdot \sqrt{(x_R - d)^2 + {y_R}^2 + (z_R - h)^2} \quad (1)$$

Where (x_R, y_R, z_R) are the coordinates of reflecting point R which can be calculated using the direction of the incident GPS signal. In order to relate the change of the geometric path length difference between the direct and reflected signals to slope movement, take the partial derivative of *L* with respect to *d* and q, i.e. $\partial L / \partial d$ and $\partial L / \partial q$. Then the relationship can be expressed as:

$$\Delta L = \Delta d\,(\partial L/\partial d) + \Delta q\,(\partial L/\partial q) \qquad (2)$$

5.2 The change of multipath and the movement of slope

The effect of multipath is characterized by four parameters (all of which are relative to the direct signal): 1) amplitude; 2) time delay; 3) phase; and 4) phase rate of change. For this analysis only 1) and 3) are significant.

Now consider first the case in which only one reflected signal from a planar surface interferes with the direct line-of-sight carrier signal. The direct and the reflected signals can be expressed as

$$\begin{aligned} S_0 &= A\cos j^{p} \\ S_1 &= a^{p} A\cos(j^{p} + q^{p}) \end{aligned} \qquad (3)$$

where, S_0 is the direct signal from the satellite and S_1 is the reflected signal from the surface; *A* and j^{p} are the amplitude and the phase of the signal, respectively; a^p is the amplitude reduction factor which fulfills the inequality $0<a^p<1$; q^p is the phase shift caused by multipath. To avoid confusion of notations with the previous discussions, a “p” in superscript is used to denote variables specific to multipath once inside the receiver.

Suppose the geometric path length difference between the direct and reflected signals is L and the microwave refractive index at the antenna site is n. L can be linked to slope parameters using Eqs. (5.12) (the 2D model) or (5.20) (the 3D model). The phase shift can be written in radians as

$$q^p = \frac{2pnL}{l} \qquad (4)$$

where l is the length of the code (pseudo-range) or wavelength (carrier phase) of the signal.

When the two signals interfere at the antenna centre, the composite signal is (e.g., Leick, 1995)

$$S = S_0 + S_1 = A'\cos(j^p + y^p) \qquad (5)$$

where the resultant amplitude A' is

$$A' = A(1 + 2a^p\cos q^p + (a^p)^2)^{1/2} \qquad (6)$$

and the phase delay y^p is

$$y^p = \tan^{-1}\left(\frac{a^p\sin q^p}{1 + a^p\cos q^p}\right) \qquad (7)$$

For the planar surface, the multipath error can be found by

$$MP = \frac{y^p}{2p}l \qquad (8)$$

Because in Eq. (7) both q^p and $2Np + q^p$ (N is an integer) will give the same y^p, there is a problem with the integer ambiguity N when trying to link multipath MP with L using Eqs. (8), (7) and (4). To address this problem, Eq. (4) is rewritten as

$$q^p + 2Np = \frac{2pnL}{l} \qquad (4a)$$

Suppose there is still another planar surface to reflect the GPS signals, and the reflected signal is

$$S_2 = a^p{}_1 A\cos(j^p + q^p{}_1) \qquad (9)$$

By introducing, $j^p{}_{new} = j^p + y^p$; $q^p{}_{new} = q^p{}_1 - y^p$; and

$$a^p{}_{new} = a^p{}_1/(1 + 2a^p\cos q^p + (a^p)^2)^{1/2}$$

Eqs. (3) and (9) can be re-written as

$$\begin{aligned} S &= A'\cos j^p{}_{new} \\ S_2 &= a^p{}_{new}A'\cos(j^p{}_{new} + q^p{}_{new}) \end{aligned} \qquad (10)$$

Hence, the new composite signal can be written as

$$\begin{aligned} S_m &= S_0 + S_1 + S_2 \\ &= (S_0 + S_1) + S_2 \\ &= A'_m\cos(j^p{}_{new} + y^p{}_m) \end{aligned} \qquad (11)$$

where A'_m and $y^p{}_m$ can be calculated using equations similar to Eqs. (6) and (7).

Therefore, an “equivalent ideal slope” with planar surface can always be found for the practical slopes which can reflect the GPS satellite signals to the antenna from multiple points. The following discussion on the single reflected signal case are equally valid for the multiple reflected signal case.

To avoid the integer ambiguity as describe in Eq. (4a), the change of multipath rather than the multipath itself is used to estimate the slope movement. The change of multipath can be expressed as

$$\begin{aligned} \Delta MP &= \Delta d \bullet (\partial MP/\partial d) + \Delta q \bullet (\partial MP/\partial q) \\ &= \Delta d \bullet PMP_{\mathrm{d}} + \Delta q \bullet PMP_{\theta} \end{aligned} \qquad (12)$$

where the partial derivatives PMP_{d} and PMP_{θ} can be calculated from Eqs. (8), (7), (4a) and (1). Note also that the integer ambiguity is not present in Eq. (12) because of taking derivative on Eq. (4a) with respect to L, which indicates that *ambiguity resolution has been avoided by using the change of multipath rather than the multipath itself*. Hence, using this equation the change of multipath ΔMP can be related to the movement of slope Δd and Δq. *Note that if* q *is held as zero, this technique can also be used for applications such as ground subsidence monitoring due to mining or underground water extraction.*

5.3 Least square estimation of slope movement using the change of multipath

Because there are at least four GPS satellites (even more than eight if low-elevation angle ones are also considered) in the field of view of an antenna anywhere on the Earth, but there are only two variables of slope movement to be estimated, there are many redundant measurements. Consider the vector of unknowns or the slope movement vector

$$\mathbf{X} = [\Delta d(t) \quad \Delta q\ (t)]^T \tag{13}$$

Hence, least square estimation is introduced into the data analysis. Denote the observation vector which consists of the change of both pseudo-range ($\Delta MP^i(t)$) and carrier phase ($\Delta MP^{phi}(t)$) multipath calculated by Eq. (12) for satellites PRN1 to PRNi all available at time *t* as

$$\mathbf{L} = [\Delta MP^{PRN1}(t) \ \ \Delta MP^{PRN2}(t) \ \ \ldots \ \Delta MP^{PRNi}(t) \\ \Delta MP^{phPRN1}(t) \ \ \Delta MP^{phPRN2}(t) \ \ldots \Delta MP^{phPRNi}(t)]^T \tag{14}$$

$$\mathbf{B} = \begin{bmatrix} PMP_d{}^{PRN1}(t) & PMP_q{}^{PRN1}(t) \\ PMP_d{}^{PRN2}(t) & PMP_q{}^{PRN2}(t) \\ \cdots\cdot & \\ PMP_d{}^{PRNi}(t) & PMP_q{}^{PRNi}(t) \\ PMP^{ph}{}_d{}^{PRN1}(t) & PMP^{ph}{}_q{}^{PRN1}(t) \\ PMP^{ph}{}_d{}^{PRN2}(t) & PMP^{ph}{}_q{}^{PRN2}(t) \\ \cdots\cdot & \\ PMP^{ph}{}_d{}^{PRNi}(t) & PMP^{ph}{}_q{}^{PRNi}(t) \end{bmatrix} \tag{15}$$

Then the coefficient matrix in Eq. (15) can be determined the same as in Eq. (12). Assume the vector of observation errors is Δ, then the observation equation is

$$\mathbf{L = B\ X +} \ \Delta \tag{16}$$

Therefore, the least square estimation of the slope movement is

$$\hat{X} = \mathbf{(B^T P\ B)^{-1}\ B^T P\ L} \tag{17}$$

where $\mathbf{P} = D_\Delta^{-1}$ and D_Δ is the variance matrix of Δ.

6 Concluding Remarks

A technique is proposed for the detection of multipath change as a signal at permanent GPS stations based on an adaptive filter using the least-mean-square algorithm. The technique was tested on some experimental data, indicating that the multipath change below the receiver noise level cannot be detected. A further test was done with some CGPS data from the Japanese GEONET when there was a snowfall. The results indicate that if there is a change in the antenna environment the proposed technique will indeed detect it in both the pseudo-range and carrier phase data.

As an application of the technique, the mathematical models for slope stability monitoring using the change of GPS multipath effects have been given. *It is found that an "equivalent ideal slope" with planar surface can always be found for the practical slopes which can reflect the GPS satellite signals to the antenna from multiple points. Moreover, ambiguity resolution has been avoided by using the change of multipath rather than the multipath itself in the monitoring of slope movement.* The change of multipath is used in a least square process to estimate the slope movement parameters by employing the mathematical models developed. The technique is designed to function as an early warning system through monitoring the precursors of a slope failure. It also provides an easy-to-implement quality assurance tool for CGPS antenna environment sensing after disasters such as typhoons, cyclones, and earthquakes. Other possible applications include the monitoring of ground subsidence.

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