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SELECTION AND SCALING OF SIMULTANEOUS BASELINES FOR GPS NETWORK ADJUSTMENT, OR CORRECT PROCEDURES FOR PROCESSING TRIVIAL BASELINES

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ABSTRACT

Static GPS observations collected in a session can be processed in either of two modes. One mode requires that all observations are processed simultaneously in a multi-baseline procedure which determines the R-1 independent baselines (R is the number of receivers involved in this session). In this mode the remaining trivial baselines are derived from the independent ones. The other mode involves the processing of single baselines. There are a total of R(R-1)/2 possible baselines, some of which may no longer be linearly dependent due to neglect of the between-baseline correlation and the fact that the data may not be collected simultaneously at all receivers. Most commercial GPS processing software are only capable of single baseline processing. Hence, many analysts still select a set of R-1 independent baselines for input into the subsequent network adjustment. However, different coordinate results are likely for different selections of the independent baselines. Some analysts suggest that all baselines should be selected and that their co-factor matrices be scaled by some quantity, such as factor R/2. To date no expression for the unit weight standard deviation for such a session adjustment has been derived. In this paper, the authors give a derivation why co-factor matrices should be scaled by R/2 and a formula for the computation of the unit weight standard deviation, and discuss the conditions under which it is applicable. In addition, the algorithm is extended to consider between-epoch correlations in the observations. Numerical results are presented to support the conclusions.

1. INTRODUCTION

GPS surveys can be considered to involve two steps. The first step is the single session adjustment of simultaneous observations made by a number of receivers to determine the baseline vectors. The second step is a network adjustment procedure to determine station coordinates using the baseline vector information derived in the first step as input data. Static GPS data collected in a session can be processed in one of two modes. One mode involves the simultaneous processing of all observations in a so- called "multi-baseline determination" of the R-1 independent baselines (the remaining trivial baselines can be derived from the independent ones). Any set of R-1 independent baselines can then be selected for input into the network adjustment and the same results will be obtained. These days only scientific GPS software used for very precise geodetic applications employ the multi-baseline processing strategy. The other mode is used by commercial GPS processing software and involves the reduction of data from a single pair of receivers. In a session involving R receivers, there are R(R-1)/2 possible baselines. The selection of baselines in a session is therefore a practical problem in GPS surveying.

Although single baseline processing or multi-baseline processing give the same baseline vectors for an exactly simultaneous session (i.e. all receivers commencing and stopping data collection at the same time), and hence the loop misclosure is zero (see eqn (24) in this paper), the covariance matrix is not the same due to the omission of the between-baseline correlation when data is processed in the single baseline mode. In general, different receivers in a session will not collect exactly simultaneous data, hence loop closure tests on baselines determined with different spans of data will not be zero. Many analysts suggest that a set of R-1 independent baselines should be selected for input to the network adjustment, and the remainder be discarded. A consequence of this is that the outcome of a network adjustment depends on which set of R-1 baseline vectors has been chosen, and hence the solution is no longer unique. Other analysts suggest that all baselines (independent and trivial) should be selected. However, network adjustment using all session baselines will distort the formal accuracies by artificially increasing the redundancy in the model, resulting in overly optimistic variance-covariance (VCV) matrices. Beutler et al. (1987), Beck et al. (1989), Hollmann et al. (1990) show that there is little difference in the network coordinate values when correlations are ignored, or whether all baseline combinations are used. On the other hand, Vincenty (1987), Craymer et al. (1990, 1992) and Jivall (1992) claim that incorporating all possible baseline solutions with the co-factor matrices scaled by R/2, is mathematically equivalent to a multi-baseline session adjustment under certain conditions, if the constant unit weight variance is assumed the same for possible baselines. But the *a posteriori* unit weight variances for all these baselines are not the same. A more rigorous method of determining these variances is needed.

In this paper, the authors first present a derivation why co-factor matrices should be scaled by R/2, and the conditions of its applicability, and extend the conclusions for baseline results that consider the between-epoch correlations in the observations. A formula for the unit weight standard deviation is then derived. Numerical results are presented to support the conclusions.

2. MULTI-BASELINE DETERMINATION

If multi-baseline processing software is used to determine the session baselines, the selection of the simultaneous baselines for network adjustment is straightforward. However commercial GPS processing software can only process data in a single baseline mode. In order to derive a procedure for selecting independent baselines in a session, it is necessary to first review the mathematical basis of the multi-baseline procedure.

Denote the variance-covariance matrix of the n-dimensional vector ϕ_r^s as C_ϕ, where r identifies the receiver, s identifies the satellite and n is the number of epochs. (It should be noted that C_ϕ can include the between-epoch physical correlations.) The observation vector for receiver r can be formed as:

$$\phi_r = \begin{bmatrix} \phi_r^1 \\ \vdots \\ \phi_r^S \end{bmatrix} \qquad (1)$$

where S is the total number of satellites. If the between-satellite physical correlations are neglected, its variance-covariance matrix is:

$$C_{\phi_r} = I_{S\times S} \otimes C_\phi \qquad (2)$$

Where $I_{S\times S}$ is the S×S unit matrix, and $\otimes$ is the Kronecker product which is defined as $A_{m\times n} \otimes B_{l\times k} = (a_{i,j} \cdot B)_{m\cdot l\times n\cdot k}$, where $A_{m\times n} = (a_{i,j})_{m\times n}$. The between-satellite difference operator using satellite 1 as the reference or base satellite can be defined as:

$$\Delta_m = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ -1 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & 0 & 0 & \cdots & 1 \end{bmatrix}_{(m-1)\times m} \tag{3}$$

The difference operator for S satellites, with satellite 1 as the reference satellite, can therefore be written as:

$$D_S = \Delta_S \otimes I_{n\times n} \tag{4}$$

where $I_{n\times n}$ is the n×n unit matrix and n is the number of epochs. The single difference between-satellite observation vector for receiver r is:

$$\nabla\phi_r = \begin{bmatrix} \phi_r^2 - \phi_r^1 \\ \vdots \\ \phi_r^S - \phi_r^1 \end{bmatrix} = D_S \phi_r \tag{5}$$

The variance-covariance matrix therefore is:

$$C_{\nabla\phi_r} = D_S C_{\phi_r} D_S^T = (\Delta_S \Delta_S^T) \otimes C_\phi = C \tag{6}$$

Here C is a newly defined quantity. Define the single difference observation vector for all receivers as:

$$\nabla\phi = \begin{bmatrix} \nabla\phi_1 \\ \vdots \\ \nabla\phi_R \end{bmatrix} \tag{7}$$

and its variance-covariance matrix:

$$C_{\nabla\phi} = \begin{bmatrix} C_{\nabla\phi_1} & 0 & \cdots & 0 \\ 0 & C_{\nabla\phi_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_{\nabla\phi_R} \end{bmatrix} = I_{R\times R} \otimes (\Delta_S \Delta_S^T) \otimes C_\phi = I_{R\times R} \otimes C \quad (8)$$

where R is the total number of receivers. The between-receiver difference operator is:

$$D_R = \Delta_R \otimes I_{S\times S} \otimes I_{n\times n} \quad (9)$$

Δ_R has the same structure as Δ_m (eqn (3)). The double difference observation vector can be formed with receiver 1 fixed as the reference or base receiver:

$$\Delta\nabla\phi = \begin{bmatrix} \Delta\nabla\phi_{1,2} \\ \vdots \\ \Delta\nabla\phi_{1,R} \end{bmatrix} = \begin{bmatrix} \nabla\phi_2 - \nabla\phi_1 \\ \vdots \\ \nabla\phi_R - \nabla\phi_1 \end{bmatrix} = D_R \nabla\phi \quad (10)$$

The *a priori* variance-covariance matrix therefore is:

$$C_{\Delta\nabla\phi} = D_R C_{\nabla\phi} D_R^T = (\Delta_R \Delta_R^T) \otimes C \quad (11)$$

For multi-baseline processing, the observation equation system can be written as:

$$\begin{bmatrix} V_2 \\ V_3 \\ \vdots \\ V_R \end{bmatrix} = \begin{bmatrix} A_2 & 0 & \cdots & 0 \\ 0 & A_3 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_R \end{bmatrix} \begin{bmatrix} \delta X_2 \\ \delta X_3 \\ \vdots \\ \delta X_R \end{bmatrix} - \begin{bmatrix} \Delta\nabla\phi_{1,2} - \Delta\nabla\phi_{1,2}^{calc} \\ \Delta\nabla\phi_{1,3} - \Delta\nabla\phi_{1,3}^{calc} \\ \vdots \\ \Delta\nabla\phi_{1,R} - \Delta\nabla\phi_{1,R}^{calc} \end{bmatrix} \quad (12)$$

In order to derive the relation between results of multi-baseline and those of single baseline processing, it is assumed that $A_2 = A_3 = \cdots = A_R = \overline{A}$ which is a valid approximation of the design matrix at the 0.1% level for 20 km baselines. An iterative process will ensure δX is at the submetre level and hence the above approximation will cause the coordinates to only have biases at the submillimetre level, for 20km baselines. The following results can be obtained using the above approximation:

$$\delta X_r = (\overline{A}^T C^{-1} \overline{A})^{-1} \overline{A}^T C^{-1} (\Delta\nabla\phi_{1,r} - \Delta\nabla\phi_{1,r}^{calc}) \quad (13)$$

$$Q_X = (\Delta_R \Delta_R^T) \otimes (\overline{A}^T C^{-1} \overline{A})^{-1} \tag{14}$$

$$V^T C_{\Delta\nabla\phi}^{-1} V = \begin{bmatrix} V_2^T & V_3^T & \cdots & V_R^T \end{bmatrix} [(\Delta_R \Delta_R^T)^{-1} \otimes C^{-1}] \begin{bmatrix} V_2 \\ V_3 \\ \vdots \\ V_R \end{bmatrix} = \frac{1}{R}\Omega \tag{15}$$

where

$$\Omega = R\sum_{i=2}^{R}(V_i^T C^{-1} V_i) - \sum_{i=2}^{R}\sum_{j=2}^{R} V_i^T C^{-1} V_j \tag{16}$$

and the unit weight variance is computed by:

$$\hat{\sigma}_0^2 = \frac{V^T C_{\Delta\nabla\phi}^{-1} V}{(R-1)[(S-1)n-3]} \tag{17}$$

3. SINGLE BASELINE DETERMINATION

For single baseline processing, every baseline can be computed using double differenced observation data, while the correlation between double differenced observations for different baselines are neglected. For every double differenced observation vector for a pair of GPS receivers, the following observation equation can be written if the position of receiver i is fixed:

$$V_{i,j} = A_j \delta X_{i,j} - (\Delta\nabla\phi_{i,j} - \Delta\nabla\phi_{i,j}^{calc}) \tag{18}$$

where i=1, 2, ..., R-1 and j=i+1, i+2, ..., R; $\delta X_{i,j}$ is the estimated correction value of the position of receiver j when the position of receiver i is fixed. Its *a priori* variance-covariance matrix should be (note that the between-satellite correlation is accounted for):

$$C_{\Delta\nabla\phi_{i,j}} = 2(\Delta_S \Delta_S^T) \otimes C_\phi = 2C \tag{19}$$

The baseline results can be obtained as follows:

$$\delta X_{i,j} = (A_j^T C^{-1} A_j)^{-1} A_j^T C^{-1} (\Delta\nabla\phi_{i,j} - \Delta\nabla\phi_{i,j}^{calc}) \quad (20)$$

$$Q_{X_{i,j}} = 2(A_j^T C^{-1} A_j)^{-1} \quad (21)$$

$$V_{i,j}^T C_{\Delta\nabla\phi_{i,j}}^{-1} V_{i,j} = \frac{1}{2} V_{i,j}^T C^{-1} V_{i,j} \quad (22)$$

$$\hat{\sigma}_{i,j}^2 = \frac{V_{i,j}^T C_{\Delta\nabla\phi_{i,j}}^{-1} V_{i,j}}{(S-1)n-3} \quad (23)$$

Making the same assumption $A_2 = A_3 = \cdots = A_R = \overline{A}$, for exactly simultaneous data in a session, it can be proven that:

$$\delta X_{i,j} = \delta X_{1,j} - \delta X_{1,i} \quad (24)$$

$$V_{i,j} = V_{1,j} - V_{1,i} \quad (25)$$

4. NETWORK ADJUSTMENT

4.1 Selection of the Simultaneous Baselines

If all baselines are selected in a session network adjustment, the observation equation system can be written as follows (from eqn (24)):

$$\delta X_{\cdot,\cdot} = B\delta X_{1,\cdot} \quad (26)$$

where

$$\delta X_{\cdot,\cdot} = \begin{bmatrix} \delta X_{1,2} \\ \delta X_{1,3} \\ \delta X_{1,4} \\ \delta X_{2,3} \\ \delta X_{2,4} \\ \delta X_{3,4} \end{bmatrix} \quad (27a)$$

$$B = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ -I & I & 0 \\ -I & 0 & I \\ 0 & -I & I \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \otimes I \tag{27b}$$

$$\delta X_{1,\cdot} = \begin{bmatrix} \delta X_{1,2} \\ \delta X_{1,3} \\ \delta X_{1,4} \end{bmatrix} \tag{27c}$$

and weight matrix:

$$C_X^{-1} = \mathrm{diag}\left[Q_{X_{1,2}}^{-1} \quad Q_{X_{1,3}}^{-1} \quad Q_{X_{1,4}}^{-1} \quad Q_{X_{2,3}}^{-1} \quad Q_{X_{2,4}}^{-1} \quad Q_{X_{3,4}}^{-1}\right] \tag{27d}$$

for R=4; I is the 3×3 unit matrix. For any number of receivers (R), a similar form to eqn (27) can be obtained. The Least Squares estimate for the session network adjustment is obtained:

$$\delta X_{1,\cdot} = \begin{bmatrix} \delta X_2 \\ \delta X_3 \\ \vdots \\ \delta X_R \end{bmatrix} \tag{28}$$

from eqns (13) and (20), and

$$Q_{X_{1,\cdot}} = (B^T C_X^{-1} B)^{-1} = \frac{2}{R}(\Delta_R \Delta_R^T) \otimes (\overline{A}^T C^{-1} A^T)^{-1} = \frac{2}{R} Q_X \tag{29}$$

from eqns (14), (21) and (26). On the other hand, the residual vectors will have the following relations:

$$\begin{bmatrix} V_{1,2} \\ V_{1,3} \\ \vdots \\ V_{1,R} \end{bmatrix} = \begin{bmatrix} V_2 \\ V_3 \\ \vdots \\ V_R \end{bmatrix} \tag{30}$$

$$\sum_{i=1}^{R-1}\sum_{j=i+1}^{R} V_{i,j}^T C_{\Delta\nabla\phi_{i,j}}^{-1} V_{i,j} = \frac{1}{2}\left(\sum_{j=2}^{R} V_j^T C^{-1} V_j + \sum_{i=2}^{R-1}\sum_{j=i+1}^{R}(V_j - V_i)^T C^{-1}(V_j - V_i)\right) \tag{31}$$

from eqns (22), (25) and (30) , and then

$$\sum_{i=1}^{R-1}\sum_{j=i+1}^{R} V_{i,j}^T C_{\Delta\nabla\phi_{i,j}}^{-1} V_{i,j} = \frac{1}{2}\Omega = \frac{R}{2} V^T C_{\Delta\nabla\phi}^{-1} V \tag{32}$$

$$\hat{\sigma}_0^2 = \frac{2}{R(R-1)}\sum_{i=1}^{R-1}\sum_{j=i+1}^{R}\hat{\sigma}_{i,j}^2 \tag{33}$$

from eqns (15), (16), (17) and (23). Therefore, we can see that the unit weight variance for a multi-baseline determination is the *mean value* of the unit weight variances of all the baselines in a session (eqn (33)).

Hence, if the session adjustment is carried out using co-factor matrices scaled by R/2, the baseline results, the resultant co-factor matrix and the mean value of the unit weight variances is the same as that obtained from multi-baseline processing.

4.2 Standardization of the Variance-Covariance Matrix

Although equivalent results can be obtained by scaling the co-factor matrices by R/2 and re-computing the unit weight variance as described above, there are still problems. It is well known that the VCV matrix output by baseline processing software is overly optimistic, mainly due to the fact that between-epoch correlations are neglected (Han & Rizos, 1995; El-Rabbany, 1994; Hollmann et al., 1990; Vincenty, 1987). At present no commercial GPS processing software considers these correlations. Han & Rizos (1995) describe a procedure for standardizing the variance-covariance matrix, by scaling the co-factor matrix by a quantity that is a function of the correlation coefficient between the neighboring epochs of data and the number of epochs, and recomputing the unit weight variance using residual information output by commercial GPS processing software (IBID, 1995):

$$\overline{Q}_X = \alpha \cdot Q_X \tag{34}$$

$$\hat{\sigma}_0^2 = \frac{\Omega}{r} \tag{35}$$

where

$$\alpha = \frac{n(1+f)}{n(1-f)+2f} \tag{36}$$

$$\Omega = \frac{1}{1-f^2}\sum_{i=1}^{n-1}\left[\left(\hat{V}_i^T - f\hat{V}_{i+1}^T\right)C_0^{-1}\left(\hat{V}_i - f\hat{V}_{i+1}\right)\right] \tag{37}$$

and n is the number of epochs; here r is the degree of freedom; f is the correlation coefficient between neighboring observation epochs (Han & Rizos, 1995; El-Rabbany, 1994). $\hat{V}_i$ is the residual vector at epoch i from observation processing that neglects the between-epoch correlations. This procedure is suitable for both single baseline processing and multi-baseline processing. α is independent of the number of simultaneous baselines. In the exactly simultaneous case, α is the same for both single baseline processing and multi-baseline processing.

Any model of the physical correlations within a set of single differenced observations for a pair of satellites between epochs can be considered in the *a priori* VCV matrix C_ϕ. If an appropriate exponential covariance function is suggested, the scale factor α can be applied to both sides of equation (29), and equations (35) and (37) can be used to compute the unit weight variance, hence maintaining the relationship in equation (33). Note that the smaller the data sample interval, the better approximation the equation (37) relation becomes. Normally, the data sample interval for GPS (rapid) static positioning is less than 1 minute and equation (37) will be a very good approximation.

4.3 Network Adjustment Procedures

Based on the above derivations, the following procedure for network adjustment is suggested:

(1) All possible combinations among receivers should be processed as baseline solutions, using exactly simultaneous data. Although the trivial baselines can be determined from

the independent baseline solutions by taking appropriate linear combinations, the variance-covariance information is incorrect due to the omission of the correlations between baselines.

(2) All co-factor matrices of baseline solutions in a session should be scaled by R/2 and α. The unit weight variances should be re-computed using equation (37) and the mean value of the re-computed unit weight variances for the baselines in a session should be considered as the unit weight variance for all baselines in the session. Given the scaled co-factor matrices and the unit weight variance for the session, the *a priori* VCV matrices can be derived.

(3) For the network adjustment, all baselines with their *a priori* VCV matrices are treated as baseline vector *observations*. A Least Squares adjustment can then be carried out with minimal constraint (i.e. one station held fixed).

To the above procedure, the following comments can be made:

- The above procedure is suitable for exactly simultaneous baselines. If some double difference data series have outliers, and are deleted during a baseline solution, the data for the same time interval should be deleted for all other baseline determinations as well. Although this will make the process more complicated, this ensures that the results that are obtained are numerically equivalent to the mathematically rigorous multi-baseline solution. If the data that is not completely simultaneous is used, the above procedure can therefore only be considered approximate.

- Only one point is held fixed in the network adjustment, rather than all available "known" points, for two reasons. Firstly, this provides a minimally constrained adjustment that permits the examination of the GPS only results without any distorting influence from the existing control. Any problems with this solution are therefore due to GPS, and cannot be attributed to the existing control network. The second reason is that the VCV for GPS solutions and the existing control may be incompatible. If a combination of the GPS network and the existing control network is required, the results of the GPS only network adjustment and the variance-covariance estimation should be obtained first.

- In order to ensure that the loop misclosure formed by baselines of the same session is zero, the approximate initial coordinates for each baseline should be derived from one fixed station's coordinates. This ensures that the same fixed coordinate bias affects all baseline solutions.

5. NUMERICAL EXAMPLES

Exactly simultaneous data collected by three GPS receivers during a 10 minute period (13:55:15-14:05:00) are processed. The three individual baselines are determined using the single baseline determination mode. If the co-factor matrices are scaled by R/2, but not scaled by α, the adjustment results using the three baselines with their *a priori* weight are given in Table 1a. x_1, y_1, z_1 are the adjusted coordinates of one receiver relative to the fixed receiver and x_2, y_2, z_2 are the adjusted coordinates of the other receiver. The data is also processed using the multi-baseline mode and the results of a multi-baseline solution are the same as those in Table 1a to the last digit. This demonstrates that the use of all possible baselines, with their co-factor matrices and unit weight variances, will simulate a multi-baseline solution for exactly simultaneous session. However, these results, as is obvious from Table 1a, are overly optimistic. Table 1b gives the results obtained from the three baselines using the suggested network processing procedure (Section 4.3). These are the same as the standardized results of a multi-baseline solution (to account for the between-epoch correlations), to the last digit in Table 1b, as is the unit weight variance. The last columns in Tables 1a and 1b are the *a priori* unit weight standard deviation determined from equation (33). The results in Table 1b are more realistic (note the accuracy estimation results are not overly optimistic) than the results in Table 1a. Note that the unit weight standard deviation is chosen as the standard deviation of the one-way carrier phase observation (Caspary, 1987, p94).

In order to estimate the impact of GPS data series that is not completely simultaneous, different observation spans are selected from the data set. A 10% difference in observation session length was considered. The three baselines are processed separately and the results listed in Tables 2a, 2b and 2c. Using the suggested network procedure the results of the network adjustment are given in Table 3. On comparing Table 3 and Table 1b, the baseline vector differences are less than 0.4mm and the differences in the elements of the co-factor matrix are less than 5%. The *a priori* unit weight standard deviation in the last column of

Table 3, which is the square root of the mean value of the unit weight variances of the three individual baselines, is the same as the value in Table 1b. The *a posteriori* unit weight standard deviation in the network adjustment is 0.6mm, which cannot be considered reliable due to the low degree of freedom for this adjustment.

Table 1a. Single Baseline Network Adjustment (only scaled by R/2)

	$X_{1,\cdot}$ (m)	$Q_{X_{1,\cdot}}$						$\hat{\sigma}_0$ (m)
x_1	-3277.4976	0.0352						0.0024
y_1	-2447.6895	-0.0304	0.1410					
z_1	674.6098	-0.0210	0.0938	0.1467				
x_2	-3275.1088	0.0176	-0.0152	-0.0105	0.0352			
y_2	-1452.5854	-0.0152	0.0705	0.0469	-0.0304	0.1410		
z_2	-345.5081	-0.0104	0.0469	0.0734	-0.0210	0.0938	0.1467	

Table 1b. Single Baseline Network Adjustment using the Suggested Network Procedure (Section 4.3)

	$X_{1,\cdot}$ (m)	$\overline{Q}_{X_{1,\cdot}}$						$\hat{\sigma}_0$ (m)
x_1	-3277.4976	0.6633						0.0034
y_1	-2447.6895	-0.5722	2.6558					
z_1	674.6098	-0.3952	1.7672	2.7636				
x_2	-3275.1088	0.3317	-0.2861	-0.1976	0.6633			
y_2	-1452.5854	-0.2861	1.3279	0.8835	-0.5722	2.6557		
z_2	-345.5081	-0.1976	0.8837	1.3818	-0.3952	1.7673	2.7638	

Table 2a. Baseline Determination (Stn 1 - Stn 2, Time: 13:55:15-14:04:00)

	$X_{1,2}$ (m)	$Q_{X_{1,2}}$			$\hat{\sigma}^2_{1,2}(10^{-6}m^2)$	α
x	-3277.4983	0.0390			14.2640	17.9262
y	-2447.6886	-0.0335	0.1563			
z	674.6101	-0.0230	0.1034	0.1614		

Table 2b. Baseline Determination (Stn 1 - Stn 3, Time: 13:55:15-14:05:00)

	$X_{1,3}$(m)	$Q_{X_{1,3}}$			$\hat{\sigma}^2_{1,3}(10^{-6}m^2)$	α
x	-3275.1086	0.0352			8.8718	18.8372
y	-1452.5854	-0.0304	0.1410			
z	-345.5081	-0.0210	0.0938	0.1467		

Table 2c. Baseline Determination (Stn 2 - Stn 3, Time: 13:55:15-14:06:00)

	$X_{2,3}$(m)	$Q_{X_{2,3}}$			$\hat{\sigma}^2_{2,3}(10^{-6}m^2)$	α
x	2.3885	0.0321			10.9908	19.6544
y	995.1045	-0.0278	0.1284			
z	-1020.1177	-0.0193	0.0860	0.1347		

Table 3. Network Adjustment using the Suggested Network Procedure

	$X_{1,.}$(m)	$\overline{Q}_{X_{1,.}}$						$\hat{\sigma}_0$ (m)
x_1	-3277.4980	0.6811						0.0034
y_1	-2447.6891	-0.5864	2.7281					
z_1	674.6100	-0.4038	1.8102	2.8281				
x_2	-3275.1091	0.3490	-0.3000	-0.2059	0.6640			
y_2	-1452.5850	-0.3000	1.3989	0.9256	-0.5727	2.6582		
z_2	-345.5079	-0.2059	0.9256	1.4446	-0.3955	1.7687	2.7660	

From the numerical results presented above, it has been demonstrated that the suggested network adjustment procedure for exactly simultaneous data will lead to the same results as using multi-baseline processing software, and the estimated accuracy is much more realistic. For session data which is not completely simultaneous, such as the case of a 10% difference in the observation session lengths, the loop test misclosures will not be zero. The suggested procedure can give good approximate results which are unique and ensure that the loop misclosures are zero.

6. CONCLUDING REMARKS

Least Squares adjustment requires correct *a priori* information. The standardized variance-covariance matrices give realistic descriptions of the GPS baseline accuracies. The correlations between baselines are generally omitted in commercial GPS processing software. In order to obtain equivalent results to the mathematically rigorous multi-baseline processing (in which baseline correlations are considered), all baselines in a session should be selected and processed, and the co-factor matrices scaled by R/2 (R is the number of receivers involved in this session). A unit weight variance which is the mean of all unit weight variances of simultaneous baselines in a session should then be used.

The condition to be satisfied by the session data is that the data for all baselines in a session should be exactly simultaneous. For data which is not completely simultaneous, only approximate results can be obtained using the suggested procedure. However the results remove the misclosures of simultaneous closed loop tests and ensure the network adjustment results are unique.

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REVIEWER'S COMMENT

In this paper and a previous paper (Han & Rizos, 1995), the authors chose to assign units to the a priori and a posteriori unit weight variance. This is not the preferred practice of the reviewer, or the editors.

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