The Impact of Two Additional Civilian GPS Frequencies on Ambiguity Resolution Strategies

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BIOGRAPHY

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Chris Rizos holds a B.Surv. and Ph.D., both obtained from The University of New South Wales. He has been an academic staff member of the School of Surveying (renamed the School of Geomatic Engineering in 1994) since 1987, and is presently an Associate Professor. Chris has published over 100 papers, as well as having authored and co-authored several books relating to GPS and positioning technologies. He is leader of the Satellite Navigation and Positioning (SNAP) group at UNSW, specialising in precise static and kinematic applications of the GPS technology. SNAP is considered the premier academic GPS R&D group in Australia.

ABSTRACT

In Washington D.C., on 25 January 1999, Vice President Al Gore announced that two new civil signals would be broadcast on future Block IIF GPS satellites. These new civilian signals, in combination with the L1 signal now available, will greatly enhance the accuracy, reliability and robustness of civilian GPS receivers, resulting in significantly improved system capabilities. This improved GPS performance will have a tremendous impact on navigation, positioning and timing services, but especially for high accuracy users engaged in surveying, geodesy, atmospheric monitoring, etc., who make use of GPS carrier phase measurements. The mathematical process that converts ambiguous carrier phase measurements to unambiguous ranges of millimetre measurement precision, referred as Ambiguity Resolution (AR), is a critical requirement for high accuracy positioning. AR techniques will be significantly impacted upon were two additional civilian frequencies to become available. In this paper, the AR strategies that take advantage of carrier phase measurements on the second and third signal frequencies will be discussed. The linear combination theory of carrier phase measurements will be extended, and following error analysis, several new combination measurements which have longer effective wavelength and less "noise" will be proposed. The new measurement combinations provide an opportunity to resolve double-differenced integer ambiguities using pseudo-range observations directly and with almost no baseline length constraints. Although the new signals are planned only for the Block IIF satellites, and the service will not be available before 2005, the AR strategies described in this paper may be useful for planning purposes, particularly in regard to new applications that may be addressed by lower cost, multiple frequency GPS user technology in the early years of the 21st century.

INTRODUCTION

Precise GPS positioning requires the use of carrier phase measurements, the data processing of which suffers from having to deal with the integer ambiguities. Ambiguity Resolution (AR) is the mathematical process of converting ambiguous ranges (carrier phase measurements) to unambiguous range data with millimetre precision. Many ambiguity resolution techniques using single-frequency or dual-frequency measurements have been developed over the last two decades. For some applications, e.g. static relative positioning over short distances, the ambiguity resolution techniques are quite mature and would be available to deliver centimeter accuracy positioning with high reliability. For kinematic positioning, or long-range static positioning, the integer ambiguities cannot be reliably determined, or the process suffers from many constraints. However, the unique capabilities of GPS for high precision static and kinematic positioning are highly valued for many aviation, agriculture, automotive, space systems, surveying, and other applications. Driven by these applications, The United States Vice President Al Gore announced in March 1998 that the second civil signal located at 1227.60MHz, which is now used mainly by the military (apart from users equipped with special "codeless" L2 tracking receivers), will be generally available on the satellites scheduled for launch beginning in 2003. Moreover, in January 1999, Vice President Gore announced that the third civil signal, which is to be located at 1176.45MHz, will be implemented beginning with a satellite scheduled for launch in 2005. When combined with the current L1 civil signal at 1575.42MHz, the new signals will significantly improve the robustness and reliability of GPS for civilian users, and consequently will support many new applications that benefit from carrier phase measurements on several frequencies.

The main benefits for precise GPS positioning is that triple-frequency measurements will significantly help ambiguity resolution, and hence increase the reliability of precise GPS positioning rather than positioning accuracy. Strategies for making use of the triple-frequency measurements have had been studied by Hatch (1996) and Forssel et al. (1997) even before the third signal frequency was known. In this paper, the integer linear combinations for the triple-frequency case have been investigated as an extension of similar studies on dual-frequency data combinations by Han & Rizos (1996). The possibility of ambiguity resolution for certain data combinations is then discussed assuming the implementation of the LAMBDA AR method (Teunissen, 1993, 1995; Han & Rizos, 1995). Alternative ambiguity resolution strategies, in the case of baseline length constraints being applied and without these constraints, are suggested.

LINEAR COMBINATION OF CARRIER PHASE MEASUREMENTS

The ambiguities can be determined using pseudo-range and carrier phase data directly. Unfortunately the accuracy of the C/A or P-code pseudo-ranges is not good enough, on their own, to determine the integer ambiguities because the wavelength of the carrier phase observable is only 19.03cm for L1, 24.42cm for L2 and 25.48cm for L3 (the new frequency at 1176.45MHz). Similar to the linear combination investigation for the dual-frequency case described in Han & Rizos (1996), the integer linear combinations of the L1, L2 and L3 carrier phase observations which have a relatively long wavelength, low noise characteristics, and experience a reasonably small ionospheric delay can be studied.

It is very difficult, if not impossible, to determine integer ambiguity for one-way data because they suffer from the L1, L2 and L3 clock divergence in the satellite and receiver (Hatch, 1996). Therefore, the double-differenced carrier phase ambiguities should be formed and resolved to their integer values. The fundamental measurements from modernised GPS system will be three pseudo-range and three carrier phase measurements. The observation equations can be written as:

$$R_1 = r + \frac{I}{f_1^2} + e_{R_1} \tag{1}$$

$$R_2 = r + \frac{f_1^2}{f_2^2} \cdot \frac{I}{f_1^2} + e_{R_2} \tag{2}$$

$$R_3 = r + \frac{f_1^2}{f_3^2} \cdot \frac{I}{f_1^2} + e_{R_3} \tag{3}$$

$$j_1 = \frac{r}{l_1} - \frac{1}{l_1} \cdot \frac{I}{f_1^2} + N_1 + e_{j_1} \tag{4}$$

$$j_2 = \frac{r}{l_2} - \frac{f_1^2}{f_2^2 l_2} \cdot \frac{I}{f_1^2} + N_2 + e_{j_2} \tag{5}$$

$$j_3 = \frac{r}{l_3} - \frac{f_1^2}{f_3^2 l_3} \cdot \frac{I}{f_1^2} + N_3 + e_{j_3} \tag{6}$$

where R_1, R_2 and R_3 are the double-differenced precise pseudo-ranges on L1, L2 and L3; j_1, j_2 and j_3 are the double-differenced carrier phase observations in units of cycles; r is the double-differenced geometric range from receiver to satellite; I is a function of the Total Electron Content (TEC) of the ionosphere; f_1, f_2, f_3 and l_1, l_2, l_3 are the frequencies and wavelengths of the L1, L2 and L3 carrier waves respectively; N_1, N_2 and N_3 are the integer cycle ambiguities of the L1, L2 and L3 double-differenced carrier phase observations; and *e* is the observation noise with respect to the observation type *indicated* by its subscript. The linear combination of carrier phase measurements for the triple-frequency case can be defined as (Han & Rizos, 1996):

$$j_{i,j,k} = i \cdot j_1 + j \cdot j_2 + k \cdot j_3 \tag{7}$$

and the observation equation can be derived:

$$j_{i,j,k} = \frac{r}{l_{i,j,k}} - \frac{i+77j/60+154k/115}{i+60j/77+115k/154} \cdot \frac{1}{l_{i,j,k}} \cdot \frac{I}{f_1^2} + N_{i,j,k} + e_{j_{i,j,k}} \tag{8}$$

The effective frequency, wavelength and integer ambiguity combination can be formed:

$$f_{i,j,k} = i \cdot f_1 + j \cdot f_2 + k \cdot f_3 \tag{9}$$

$$l_{i,j,k} = c / f_{i,j,k} \tag{10}$$

$$N_{i,j,k} = i \cdot N_1 + j \cdot N_2 + k \cdot N_3 \tag{11}$$

If the tropospheric delay and the orbit bias are denoted as $d_{trop,orbit}$ and considered as error sources, they can be introduced into the linear combinations of the carrier phase measurements. The effect is $d_{trop,orbit} / l_{i,j,k}$ on $j_{i,j,k}$, or $d_{trop,orbit}$ on the carrier phase range $j_{i,j,k} \cdot l_{i.j.k}$.

Random Errors

If it is assumed that the standard deviations of the random errors on the three frequencies are equal to $M_0[cy]$, expressed in units of cycles of the corresponding wavelength, the standard deviation $M[cy]$ of the linear combination is:

$$M_{i,j,k}[cy] = \sqrt{i^2 + j^2 + k^2} \cdot M_0[cy] \tag{12}$$

$$M_{i,j,k}[m] = M_{i,j,k}[cy] \cdot l_{i,j,k} \tag{13}$$

These formulae clearly show that the random error, expressed in cycles of the effective wavelength, is always greater than the noise on either L1, L2 or L3 carrier phase measurements. However, the noise level for combinations in units of meters may be smaller than the noise on either L1, L2 or L3 carrier phase measurements.

Ionospheric Delay

The ionospheric delay (in meters) on the range $j_{i,j,k} \cdot l_{i,j,k}$ can be represented as:

$$d^{ion}_{i,j,k} = K_{i,j,k} \cdot \frac{I}{f_1^2} \tag{14}$$

where $K_{i,j,k}$ is the ratio value between the ionospheric delays on the combinations (in units of meters) and the L1 carrier phase measurement, derived as follows:

$$K_{i,j,k} = \frac{i+77j/60+154k/115}{i+60j/77+115k/154} \tag{15}$$

There are many combinations without ionospheric delay effect. However, they could be derived from the three fundamental ionosphere-free combinations $j_{77,-60,0}$, $j_{154,0,-115}$ and $j_{0,24,-23}$. This means that there are opportunities to find the *optimal* ionosphere-free combination for different purposes.

Table 1. Some Typical Ionosphere-Free Carrier Phase Combinations

$j_{i,j,k}$	$f_{i,j,k}$ [MHz]	$l_{i,j,k}$	$M_{i,j,k}[cy]$	$M_{i,j,k}[m]$
$j_{77,-60,0}$	47651.34	.0063	.97617	.00614
$j_{154,0,-115}$	107322.93	.0028	1.92200	.00537
$j_{0,24,-23}$	2404.05	.1247	.33242	.04145
$j_{77,-12,46}$	52459.44	.0057	.90493	.00517
$j_{308,-24,-207}$	212241.81	.0014	3.71872	.00525
$j_{77,-468,391}$	6782.49	.0442	6.14682	.27170

For positioning purposes, the minimal variance for the ionosphere-free combination is desired, which means that the combinations have $K_{i,j,k} = 0$ and small $M_{i,j,k}[m]$. Although the standard deviation in meters for $j_{77,-12,46}$ is not a minimum, it is very close to the minimum value and has relatively small integer coefficients. For certain applications, the combinations $j_{77,-60,0}$ and $j_{154,0,-115}$ could be used because their standard deviations are also relatively small.

For ambiguity resolution purposes, the longest wavelength of the ionosphere-free combination is desired, which means that the combinations should have $K_{i,j,k} = 0$, $f_{i,j,k} = \min$ and small $M_{i,j,k}[cy] = \min$. The combination $j_{0,24,-23}$ has these characteristics. However, the standard deviation (in meters) is quite large (4.1cm) relative to its wavelength (12.5cm). The other combination $j_{308,-24,-207}$ has small standard deviation (0.5cm) and an effective wavelength of 10.87cm when the $N_{0,1,-1}$ is fixed to its integer value. Although the combination $j_{77,-468,391}$ has quite large standard deviation (27.2cm), the effective wavelength is 3.4m when the $N_{0,1,-1}$ ambiguity is fixed. If both $N_{0,1,-1}$ and $N_{1,-6,5}$ (or $N_{1,-1,0}$) are fixed to their integer values, the integer ambiguity $N_{77,-468,391} = -6N_{0,1,-1} + 77N_{1,-6,5}$

for the ionosphere-free combination $j_{77,-468,391}$ is then fixed. From the discussion in the next section, it will be seen that $N_{0,1,-1}$ and $N_{1,-6,5}$ (or $N_{1,-1,0}$) could be easily fixed to their integer values using pseudo-range measurements.

Wavelength

It can be proven from equation (9) that the minimum frequency among all combinations is 10.23MHz. Many different combinations with the minimum frequency (or maximum wavelength) can be found, in which $j_{-6,1,7}$ has the minimum noise and $j_{-1,8,-7}$ has the minimum ionospheric effect. The second maximum wavelength combination with minimum noise is $j_{3,0,-4}$. The third and fourth with minimum noise are $j_{-3,1,3}$ and $j_{1,-7,6}$. The fifth one $j_{0,1,-1}$ is the widelane combination for L2 and L3 carrier phase, which has small noise, small ionospheric effect and long wavelength. The other two widelane combinations are $j_{1,-1,0}$, $j_{1,0,-1}$. The combination with quite long wavelength (e.g. $> l_{1,0,-1}$) and minimum ionospheric delay is $j_{1,-6,5}$.

Table 2. Some Typical Carrier Phase Combinations with Long Wavelengths

$j_{i,j,k}$	$f_{i,j,k}$ [MHz]	$l_{i,j,k}$ [m]	$M_{i\,j,k}$ [cy]	$M_{i\,j,k}$ [m]	$K_{i,j,k}$
$j_{-6,1,7}$	10.23	29.305	0.093	2.718	717.22
$j_{-1,8,-7}$	10.23	29.305	0.107	3.129	-16.52
$j_{3,0,-4}$	20.46	14.653	0.050	0.733	-180.45
$j_{-3,1,3}$	30.69	9.7684	0.044	0.426	118.1
$j_{1,-7,6}$	40.92	7.3263	0.093	0.679	1.98
$j_{0,1,-1}$	51.15	5.861	0.014	0.083	-1.72
$j_{1,-6,5}$	92.07	3.256	0.079	0.256	-0.07
$j_{1,-1,0}$	347.82	0.862	0.014	0.012	-1.28
$j_{1,0,-1}$	398.97	0.751	0.014	0.011	-1.34

AMBIGUITY RESOLUTION WITHOUT DISTANCE CONSTRAINTS

An important issue is which combinations should be used for ambiguity resolution? The real-valued ambiguities could be estimated using equations (1-6). If the standard deviations of the pseudo-range measurements on L1, L2 and L3 are assumed to be the same and denoted as s_R, the geometric range r and ionospheric delay term I/f_1^2 can be estimated from equations (1-3) using the Least Squares technique:

$$\begin{bmatrix} r \\ I/f_1^2 \end{bmatrix} = \begin{bmatrix} 2.32694 & -0.35965 & -0.96730 \\ -1.34697 & 0.46821 & 0.87876 \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} \quad (16)$$

and the integer ambiguities can be derived:

$$\begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} = \begin{bmatrix} j_1 \\ j_2 \\ j_3 \end{bmatrix} - \begin{bmatrix} 19.30654 & -4.35039 & -9.70112 \\ 18.61236 & -4.63026 & -9.88727 \\ 18.61029 & -4.70618 & -9.97990 \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} \quad (17)$$

If the standard deviation of L1, L2 and L3 carrier phase measurements are the same and denoted as s_j, the variance-covariance matrix of real-valued ambiguities can be derived:

$$D_N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot s_j^2 + \begin{bmatrix} 48578030 & 47540131 & 47659032 \\ 47540131 & 46561720 & 46684613 \\ 47659032 & 46684613 & 46808941 \end{bmatrix} \cdot s_R^2 \quad (18)$$

$$D_N = \begin{bmatrix} 48578030 + s_j^2/s_R^2 & 47540131 & 47659032 \\ 47540131 & 46561719 + s_j^2/s_R^2 & 46684613 \\ 47659032 & 46684613 & 46808941 + s_j^2/s_R^2 \end{bmatrix} \cdot s_R^2 \quad (19)$$

It is difficult to estimate the original integer ambiguities N_1, N_2 and N_3 because their variances are too large. Therefore three integer linear combinations of N_1, N_2 and N_3 should be formed:

$$\overline{N} = Z \cdot N \quad (20)$$

$$D_{\overline{N}} = Z D_N Z^T \quad (21)$$

In order to ensure that the transformed ambiguity has integer characteristics, the transformation matrix Z has to have integer entries. In order to ensure that the original ambiguity can be determined from the transformed ambiguity, the inverse of the transformation matrix must also have integer entries. Therefore, *matrix Z is an admissible ambiguity transformation if and only if matrix Z has integer entries and its determinant equals 1* (Teunissen, 1993, 1995). The original ambiguities are transformed:

$$N = Z^{-1} \cdot \overline{N} \quad (22)$$

The criteria for this integer transformation should be that the multiplication of the variances of the three integer linear combinations (or the determinant of the diagonal matrix of the variance-covariance matrix $D_{\overline{N}}$) is a minimum or, in other words, the three integer linear combinations should be highly uncorrelated. The transformation matrix Z has been determined using the algorithm proposed by Han & Rizos (1995):

$$Z = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -3 & 2 \\ 25 & -159 & 133 \end{bmatrix} \qquad (23)$$

Where it is assumed that $s_j = 0.01$ cycles and $s_R = 0.3$ meters. The Z matrix is only affected by the ratio value s_j / s_R in cycle/meter, rather than either s_j or s_R.

$$\begin{bmatrix} \overline{N}_1 \\ \overline{N}_2 \\ \overline{N}_3 \end{bmatrix} = Z \cdot \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} = \begin{bmatrix} N_{0,1,-1} \\ N_{1,-3,2} \\ N_{25,-159,133} \end{bmatrix} \qquad (24)$$

For this case $D_{\overline{N}}$ can be determined:

$$D_{\overline{N}} = Z \cdot D_N \cdot Z^T = \begin{bmatrix} 0.00149 & 0.00051 & -0.00052 \\ 0.00051 & 0.04573 & -0.00058 \\ -0.00052 & -0.00058 & 5.22540 \end{bmatrix} \qquad (25)$$

The new integer ambiguity set $\overline{N}_1$, $\overline{N}_2$ and $\overline{N}_3$ have the minimal standard deviations. The corresponding optimal linear combinations of the L1, L2 and L3 carrier phase measurements are:

$$Z \cdot \begin{bmatrix} j_1 \\ j_2 \\ j_3 \end{bmatrix} = \begin{bmatrix} j_2 - j_3 \\ j_1 - 3 \cdot j_2 + 2 \cdot j_3 \\ 25 \cdot j_1 - 159 \cdot j_2 + 133 \cdot j_3 \end{bmatrix} = \begin{bmatrix} j_{0,1,-1} \\ j_{1,-3,2} \\ j_{25,-159,133} \end{bmatrix} \qquad (26)$$

with the corresponding wavelengths 5.861, 1.221 and 0.451 meters, respectively. It is assumed that $N_{0,1,-1}$ can be easily fixed to the nearest integer cycle value because its standard deviation is very small (0.039 cycles). The other two, $N_{1,-3,2}$ and $N_{25,-159,133}$, are not easily fixed as their standard deviations are 0.214 cycles and 2.286 cycles, respectively. It is assumed that the integer value for $N_{0,1,-1}$ is fixed as known. A similar procedure to that above can be applied to determine the other two integer linear combinations. The results are denoted as $\begin{bmatrix} N_{1,-1,0} \\ N_{25,-26,0} \end{bmatrix}$ and their variance-covariance matrix $\begin{bmatrix} 0.04556 & -0.00040 \\ -0.00040 & 5.22522 \end{bmatrix}$. The wavelengths for the corresponding integer linear combinations are 0.862 and 0.040 meters, respectively. This means that it still hard to fix the integer ambiguities for the optimal combinations $\begin{bmatrix} N_{1,-1,0} \\ N_{25,-26,0} \end{bmatrix}$. If both $N_{0,1,-1}$ and $N_{1,-1,0}$ are fixed to their integer values, the $N_{1,0,0}$ could be estimated with a standard deviation of 2.286.

In conclusion, no matter how long the baseline is, $N_{0,1,-1}$ can be fixed using pseudo-range measurements directly, which means that the widelane carrier phase measurements of L2 and L3 are always available without ambiguity. They could be used for positioning with standard deviation of 7cm (assuming $s_j = 0.01$) and ionospheric effect $-1.74 \cdot I / f_1^2$ in meters. However, the integer ambiguities for the other two combinations (which cannot be derived from $N_{0,1,-1}$) cannot be determined instantaneously. An averaging process has to be implemented to reduce the noise. It is very likely that $N_{1,-1,0}$ could be fixed over a short period, e.g. several epochs. However, it is harder to fix $N_{1,0,0}$ because its standard deviation is 2.28 cycles. Theoretically 523 epochs are required to reduce the standard deviation to 0.1 cycles if the correlation between epochs is ignored. (Obviously the data rate will also affect the performance of the averaging procedure because the correlation coefficient is dependent on the data rate.) On the other hand, after $N_{0,1,-1}$ and $N_{1,-1,0}$ are fixed, the integer ambiguity $N_{77,-468,391}$ for the ionosphere-free combination $j_{77,-468,391}$ can be computed from $N_{0,1,-1}$ and $N_{1,-1,0}$. This ionosphere-free combination can be used for positioning or to fix the integer ambiguity for the other ionosphere-free combination $j_{0,24,-23}$ using an averaging procedure over time. In theory 487 epochs are needed if the standard deviation of $N_{0,24,-23}$ is required to be reduced to less than 0.1 cycles.

This technique can be used to fix integer ambiguities based on the double-differenced measurements for a pair of satellites and a pair of receivers without imposing any requirement on the baseline length, and is independent of the other pairs of satellites and receivers. The technique can then be used for a wide range of GPS positioning applications in kinematic or static mode. Although the tropospheric delay and orbit biases will affect the positioning results, the performance of the proposed ambiguity resolution technique without baseline length constraints will not be affected by these two terms because they are absorbed by the geometric range term. However, the performance of this technique suffers from measurement noise and multipath effects.

When the averaging procedure is implemented, the cycle slip issue must be addressed. However, when the triple-frequency measurements are available, the cycle slips or data gaps can be easily recovered using the long wavelength combinations with reasonably small noise and small ionospheric effect, e.g. $j_{-1,8,-7}$, $j_{3,0,-4}$, $j_{-3,1,3}$, $j_{1,-7,6}$ or $j_{0,1,-1}$ (Han, 1997).

AMBIGUITY RESOLUTION WITH DISTANCE CONSTRAINTS

If the baseline length is quite short, so that the ionospheric delay term can be ignored, the ambiguity terms can be estimated using equations (1-6) (without the ionospheric delay term). Using the same assumption that $s_j = 0.01$ cycles and $s_R = 0.3$ meters, the variance-covariance matrix is determined as:

$$D_N = \begin{bmatrix} 0.82856193 & 0.64555475 & 0.61865664 \\ 0.64555475 & 0.59312968 & 0.48207011 \\ 0.61865664 & 0.48207011 & 0.46208385 \end{bmatrix} \quad (27)$$

$$Z = \begin{bmatrix} 0 & 1 & -1 \\ -3 & 1 & 3 \\ 4 & -7 & 2 \end{bmatrix} \quad (28)$$

$$D_{\bar{N}} = \begin{bmatrix} 0.00107 & 0.00032 & 0.00015 \\ 0.00032 & 0.00221 & -0.00067 \\ 0.00015 & -0.00067 & 0.00816 \end{bmatrix} \quad (29)$$

It can seen that the integer ambiguities could be fixed, however these combinations will suffer from the ionospheric delay with the values of $0.040689 \cdot I/f_1^2$, $-12.242 \cdot I/f_1^2$ and $-11.81 \cdot I/f_1^2$ cycles. If the ionsopheric delay is also minimised under the condition that the variance of the linearly combined ambiguity is required to be less than 0.01, the linear combinations and variance-covariance matrix could be determined as:

$$Z = \begin{bmatrix} 0 & 1 & -1 \\ -3 & 1 & 3 \\ 1 & -6 & 5 \end{bmatrix} \quad (30)$$

$$D_{\bar{N}} = \begin{bmatrix} 0.00107 & 0.00032 & 0.00047 \\ 0.00032 & 0.00221 & -0.00154 \\ 0.00047 & -0.00154 & 0.00903 \end{bmatrix} \quad (31)$$

and the ionospheric delay with the values of $0.041 \cdot I/f_1^2$, $-12.242 \cdot I/f_1^2$ and $-0.432 \cdot I/f_1^2$ cycles. It is shown that only two combinations with small ionospheric effect (<8% of the ionospheric effect on the L1 signal) and small standard deviation (<0.1 cycles) can be formed, e.g. $j_{0,1,-1}$ and $j_{1,-6,5}$. It can be concluded that these two combinations' ambiguities, $N_{0,1,-1}$ and $N_{1,-6,5}$, could be fixed when the baseline length is of the order of 100km in standard environments. When $N_{0,1,-1}$ and $N_{1,-6,5}$ are fixed, the ionosphere-free combination could be formed, which is $j_{77,-468,391}$. The integer ambiguity can be fixed using the relation $N_{77,-468,391} = -6N_{0,1,-1} + 77N_{1,-6,5}$. The combination of $j_{77,-468,391}$ is then used to determine the integer ambiguity for the other ionosphere-free combination, $j_{0,24,-23}$, which can be fixed using using a certain period of data. On the other hand, the distance constraints will ensure that the other distance-dependent biases, e.g. the tropospheric and orbit biases, are small. Therefore, the geometry constraint, which is formed by four or more double-differenced measurements at a single epoch, could be used to remove the noise. Although the combination $j_{77,-468,391}$ has quite a large standard deviation (27.2cm), the noise could be reduced to the centimeter level using a certain period of data and/or geometry constraint if more than four satellites are tracked.

If the baseline length is less than about 10km or so (under normal conditions), the ionospheric delay for $j_{-3,1,3}$, $-12.242 \cdot I/f_1^2$, could also be ignored and the $N_{-3,1,3}$ could be fixed, and then the three original ambiguities, N_1, N_2 and N_3, could be determined. The geometry constraint formed by four or more double-differenced measurements at a single epoch will then be used to validate the results, hence significantly enhancing the reliability.

CONCLUDING REMARKS

Triple-frequency measurements provide the opportunity to resolve the integer ambiguities for the wideline combination between the L2 and L3 signals using pseudo-range measurements directly. This widelane combination with standard deviation of 7cm (assuming $s_j = 0.01$) and ionospheric effect $-1.74 \cdot I/f_1^2$ in meters, can be used for GPS positioning. The integer ambiguities of the other two combinations, e.g. the widelane between L1 and L2, and the L1 carrier phase measurement, could then be fixed using an averaging procedure over time. Although the tropospheric delay and orbit biases will affect the positioning results, the performance of the ambiguity resolution technique without baseline length constraints will not be affected by these two terms because they are absorbed by the geometric range term. However, the performance of this technique suffers from measurement noise and multipath effects. This technique could be used for a wide range of GPS static and kinematic positioning applications, including those demanding real-time implementation.

If the distance between the two receivers is restricted to be less than 100km, the integer ambiguity for the other combination, e.g. $j_{1,-6,5}$, could be fixed using pseudo-range measurements directly. The integer ambiguity for the third combination (which cannot be derived from $N_{0,1,-1}$ and $N_{1,-6,5}$) can be determined using an averaging procedure over time and also through the application of geometry constraint if more than four satellites are tracked. In this case, the tropospheric and orbit biases will affect the ambiguity resolution performance.

If the distance between the two receivers is restricted to less than 10km, the integer ambiguities for the three combinations, $j_{0,1,-1}$, $j_{1,-6,5}$ and $j_{-3,1,3}$ could be fixed using pseudo-range data directly, and then the three original ambiguities, N_1, N_2 and N_3, can be derived. If more than four satellite are tracked, the geometry constraint could then be used to enhance the reliability of the results in this triple-frequency case.

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