Network Adjustment Issues Using Mixed GPS Surveying Techniques

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ABSTRACT

Modern GPS carrier phase-based survey techniques such as "rapid static" and "stop & go" are fast becoming the standard operational modes for building local geodetic control networks or for the densification of an existing network. However, such techniques do have some challenges when used together within a single survey project. For example, these mixed-mode GPS surveys do result in incompatible (and over-optimistic) variance-covariance matrices, are subject to correlations between baselines when two or more base GPS receivers are used, and can result in biased baseline vectors due to faulty ambiguity resolution compared with 'traditional' GPS static surveying techniques (based on the analysis of observation sessions of an hour or more in length). In this paper an example is presented which shows the impact on coordinate results if the standard network adjustment procedure is used. A new network adjustment procedure is proposed, which includes:

(1) Processing all possible combinations of pairs of receivers as single baseline solutions.
(2) Scaling all variance-covariance matrices in a session by a factor that is a function of the number of GPS receivers that have been used, the observation data sampling rate and the length of the observation session.
(3) At the network adjustment step requiring that all baselines, with their apriori variance-covariance matrices, be treated as new (baseline vector) observations, and the least squares adjustment performed with minimal constraints.

A methodology based on "reliability analysis", for detecting baseline vector solution biases caused by wrong ambiguity resolution, is proposed. On the basis of this study some practical suggestions for designing mixed-mode GPS survey networks are made.

1. INTRODUCTION

GPS survey network processing always involves two steps. The first is the use of GPS data processing software to estimate the simultaneously observed independent baselines. The second step is the network reduction in which the coordinates of all network points are estimated within an adjustment using the baseline vectors as "observations". In the conventional procedure therefore the output of the first step is directly used as the input for the second step.

In the first step, the baseline accuracies will be dramatically improved if the cycle ambiguities are fixed to their integer values. The additional observations made after fixing, or resolving, ambiguities do not have a major impact on the baseline accuracy obtained. This is why GPS rapid static positioning techniques can obtain almost the same accuracies as conventional static GPS surveys, even though the observation session may be as short as 5 minutes for rapid static surveying compared with 60 minutes or more for conventional static GPS surveying. The main problem at this stage is that the formal accuracy estimate is overly optimistic and does not reflect the true baseline accuracy. The formal co-factor output is always in proportion to the data sample interval and inversely proportional to the length of the observation period. The unit weight variance, or variance factor, is not consistent with the *a priori* unit weight variance. The reason is that the observation equation model for double-differenced carrier phase observations does not include all systematic biases which are present, and these biases cause the observations to have strong temporal and spatial correlations.

Ananga et al. (1994) and Schaffrin & Zielinski (1989) suggest that the variance-covariance (VCV) matrix should be computed externally to the baseline solution, and that the output VCV matrix from the baseline solution should therefore be discarded. Craymer et al. (1990) suggest simply that the VCV matrix be scaled by 9.0! This method does not overcome the problem that the formal co-factor is highly dependent on the length of observation session and the data measurement rate. El-Rabbany (1994) proposes a physical correlation model to describe the observation statistical model and introduces a sequential least squares adjustment procedure in which the physical correlations can be considered. In this way a realistic variance-covariance matrix can be obtained. To date no commercial GPS software package has considered the physical correlations and none output a realistic variance-covariance matrix. Han & Rizos (1995a) describe the standardization procedures to be followed when using the VCV matrix output by commercial GPS software. The resulting *standardized* VCV matrix is almost identical to that computed in the case that the physical correlations are considered.

In the second step, the baseline vectors determined in the first step, together with their standardized VCV matrices, are used as observations in the subsequent network adjustment. There is a complication when more than two GPS receivers are simultaneously used.

In conventional static GPS surveying often more than two GPS receivers are deployed during one session (say R is the number of receivers) in order to improve the survey "productivity" (as measured in terms of number of baselines per day). Typically all GPS receivers are set up and collect data at the same time. At the end of the session, some of the receivers are moved to new benchmarks, and some are left on the previously occupied stations. During each session there are R-1 independent baselines (all other possible baselines are "trivial" or "non-independent"). The most mathematically rigorous data processing technique is the so-called "multi-baseline" technique, as is used for ultra precise geodetic software. If multi-baseline processing software were used to reduce the carrier phase data, we can simply select R-1 independent baseline vectors from a session solution, with their corresponding standardized VCV matrices, and unique results will be obtained no matter which set of R-1 independent baseline vectors are selected. Unfortunately commercial GPS software can only process single baselines. Hence, by omitting the correlations between the baselines, different selections of the R-1 independent baseline vectors will lead to a different result from the network adjustment. Beutler et al. (1987), Beck et al. (1989) and Hollmann et al. (1990) suggest that there is "little difference in the coordinate values" when correlations are ignored and all baseline combinations are used. On the other hand, Vincenty (1987), Craymer et al. (1990), Craymer & Beck (1992) and Jivall (1992) show that using all possible baseline solutions, with their co-factor matrices, will lead to overly optimistic results and suggest that the co-factor matrix then be scaled by R/2. In this way, the adjustment is mathematically equivalent to using co-factor matrices that had been output by a multi-baseline estimation program, *under certain conditions*. Han & Rizos (1995b) derive the unit weight variance for a session, and subsequently the *scaled* variance-covariance matrices for the network adjustment.

In the case of modern GPS surveying techniques the receiver deployment scenario (and therefore the pattern of independent baselines) is different. Typically the mobile receivers operate independently of each other, and it is unlikely that a "session" might include data from more than one independent baselines except when the two-base station mode of surveying is used (see Figure 2). In this case there are always two independent baseline into a single benchmark, one from each of the base station receivers.

In addition to the accuracy measure, the reliabilities are also important GPS network quality measures (Delikaraoglou & Lahaye, 1989). The reliabilities of single (or multi-) baseline ambiguity resolution have attracted much attention. The first rapid static positioning procedure was suggested by Remondi (1985), and is known as "stop & go". Since 1985, there have been many techniques developed, including "antenna swap" (Remondi, 1988), the extra-widelaning technique (Wübbena, 1988), the fast ambiguity resolution approach (FARA) (Frei & Beutler, 1990) and the least-squares ambiguity decorrelation adjustment approach (LAMBDA) (Teunissen, 1994).

In this paper, based on the network adjustment procedures suggested by Han & Rizos (1995b), the reliability measures for the network adjustment should be *re-estimated*, especially for GPS rapid static positioning where the length of observation session varies from a few minutes to 20 minutes (depending on the satellite constellation and the operational mode of GPS rapid static positioning that is employed). The standardization of the variance-covariance matrices will result in the relative accuracy being quite different from that obtained using the formal VCV output. Hence, the reliability measures will also be different for the same network. Using an example of a network observed using mixed GPS surveying techniques at the most recent Senior Survey Camp by final year geometaics students from UNSW, the authors discuss the impact on the network adjustment of the suggested procedures compared with the conventional approach. Based on the network quality measures, GPS rapid static network design is discussed and some practical receiver deployment strategies are suggested.

2. PROPOSED NETWORK ADJUSTMENT PROCEDURE -- THE MATHEMATICAL BASIS

2.1 Standardization of the Variance-Covariance matrix

It is well known that the VCV matrix output by baseline determination software is overly optimistic, mainly due to the neglect of physical correlations in time (Han & Rizos, 1995a; El-Rabbany, 1994; Hollmann et al., 1990; Vincenty, 1987). However, commercial GPS software does not take into account these physical correlations. Han & Rizos (1995a) present a convenient method for standardizing the VCV matrix, by scaling the co-factor matrix Q_X by a factor α which is a function of the correlation coefficient between neighbouring observation epochs and the number of epochs, and recomputing the unit weight variance using the co-factor matrix and measurement residual series output by commercial GPS software. The relevant equations are (ibid, 1995a):

$$SQ_X = a \cdot Q_X \qquad (1)$$

$$sm_0^2 = \frac{\Omega}{r} \qquad (2)$$

where

$$a = \frac{n(1+f)}{n(1-f)+2f} \tag{3}$$

and

$$\Omega = \frac{1}{1-f^2}\sum_{i=1}^{n-1}\left[\left(V_i^T - fV_{i+1}^T\right)C_0^{-1}\left(V_i - fV_{i+1}\right)\right] \tag{4}$$

SQ_X is the standardized co-factor matrix and sm_0^2 is the standardized unit weight variance; $sm_0^2 \cdot SQ_X$ is the standardized VCV matrix; n is the number of epochs; r is the degree of freedom in single or multi-baseline processing; f is the correlation coefficient between neighbouring observation epochs. V_i is the residual vector at epoch i, neglecting the correlations between epochs; C_0 is the variance-covariance matrix of the observations at epoch i. Note that the smaller the data sample interval is, the better the equation (4) approximates reality. Normally, the data sample interval for GPS rapid static positioning is less than 1 minute and equation (4) will be a very good approximation.

2.2 Baseline Selection for the Simultaneous Session

Network adjustment should use independent simultaneous baselines with their variance-covariance matrix determined by suitable multi-baseline determination software. Unfortunately, as already mentioned, commercial software packages used for (rapid) static GPS surveying are only capable of single baseline processing, and individual baselines are processed one-by-one in a sequential fashion. Consequently, the mathematical correlations between simultaneously observed baselines are neglected and hence an arbitrary set of independent baselines (which cannot be linearly combined to represent the trivial baselines) are introduced into the network adjustment. Different selections of independent baselines will influence the strength and reliability of the network in different ways.

Han & Rizos (1995b) suggest that all baselines in a session should be selected, with the standardized co-factor matrices scaled by R/2 and the unit weight variance for the session being determined from the standardized unit weight variances of the baselines in the session. The relations are:

$$Q_{X_{i,j}} = \frac{R}{2} SQ_{X_{i,j}} \tag{5}$$

where $X_{i,j}$ are all baseline vectors in a session, with standardized co-factor matrices $SQ_{X_{i,j}}$ (equation (1)); $Q_{X_{i,j}}$ are scaled co-factor matrices; $i = 1, 2, \cdots, R-1$; $j = i+1, \cdots, R$. The unit weight variance is:

$$m_0^2 = \frac{2}{R(R-1)}\sum_{i=1}^{R-1}\sum_{j=i+1}^{R} sm_{i,j}^2 \tag{6}$$

where $sm_{i,j}^2$ is the standardized unit weight variance when baseline $X_{i,j}$ is determined (equation (2)). Hence the scaled variance-covariance matrix for a baseline is:

$$C_{X_{i,j}} = m_0^2 Q_{X_{i,j}} \tag{7}$$

If all baselines are selected, with their scaled covariance matrices, for the network adjustment, equivalent results will be obtained to a network adjustment using R-1 independent baselines estimated using multi-baseline software.

The above procedure is suitable for exactly simultaneous baselines in a session (that is, all receivers start and stop data collection at the same time and experience the same data loss during observing). In the case where data has not been collected exactly simultaneously, approximate results will be obtained, with the degree of approximation depending on how incomplete is the observation session simultaneity.

2.3 Network Adjustment Procedures

The network adjustment procedures can be summarised as follows:

(1) All possible combinations of pairs of receivers must be processed as baseline solutions using, where possible, exactly simultaneous data.

(2) All co-factor matrices of the baselines in a session should be multiplied by R/2 and ??????. The unit weight variances should be re-computed using equations (2) and (4) and the mean value of the re-computed unit weight variances of the baselines in a session is adopted as the unit weight variance for all baselines in the session. Based on the scaled standardized co-factor matrices and the unit weight variance for the session, the *a priori* VCV matrices can be formed.

(3) For the network adjustment, all baselines, with their *a priori* VCV matrices, are treated as the new (baseline vector) observations, and the least squares adjustment can be performed with minimal constraints (one point fixed).

To the above procedure, we make the following comments:

- The above procedure is suitable for exactly simultaneous baselines. If some receiver data series have observation outliers and are deleted during single baseline processing, the data for the same epoch should be deleted for the all other baseline determinations. Although this will make the process more complicated, this does ensure that the results are equivalent to a rigorous multi-baseline solution. If the data is not completely simultaneous, the above procedure can be used at an approximate level. Normally, the lengths of observation periods for different baselines differ by no more than about 10%, and hence the VCV matrix will typically exhibit differences at the 5% level.

- Only one point is held fixed, rather than all available "known" points, for two reasons. Firstly, this provides a minimally constrained network adjustment that allows us to examine the GPS-only results without any influence from the existing geodetic control. The second reason is that the VCV matrices for GPS solutions and for the existing control may be incompatible. If we must combine a GPS network and an existing control network, we should use the results of the GPS-only network adjustment and the variance-covariance estimation should be done before any combination.

- In order to ensure that the exact simultaneous misclosure is zero, we must use the approximate initial coordinates derived from the coordinates of one fixed station. This means that the same fixed coordinate biases all baseline determinations.

3. THE RELIABILITY OF THE GPS NETWORK

For an introduction to data "snooping" and reliability analysis, the reader is referred to a monograph such as Caspary (1987).

3.1 Baarda's Reliability Theory

The basic observation model is:

$$L + V = A\hat{X} \tag{8}$$

where L is the observation vector, V is the vector of residuals and P is the observation weight matrix. The null-hypothesis is:

$$H_0: E(L / H_0) = A\tilde{X} \tag{9}$$

and the alternative hypothesis is:

$$H_a: E(L / H_a) = A\tilde{X} + H\tilde{\nabla} \tag{10}$$

where A and H are the design matrices for unknown parameters X and outlier parameters ∇ respectively; ~ denotes the expectation of the quantity. If the null-hypothesis is assumed to be true, the results should be:

$$\hat{X} = (A^T PA)^{-1} A^T PL \tag{11}$$

and

$$V = -(P^{-1} - A(A^T PA)^{-1} A^T)PL = -RL \tag{12}$$

where R is <u>not</u> the number of receivers. If the alternative hypothesis is assumed to be true, we can obtain:

$$\hat{\nabla} = P_{\nabla}^{-1} H^T PRL \tag{13}$$

where

$$P_{\nabla} = H^T PRH \tag{14}$$

If the unit weight variance is known, the following statistic can be obtained

$$T = \frac{\hat{\nabla}^T P_{\nabla} \hat{\nabla}}{\sigma_0^2} \sim \chi'^2 (p, \delta^2) \tag{15}$$

where p is the number of outlier parameters; δ^2 is the non-centrality parameter which can be represented by:

$$\delta^2 = \frac{\tilde{\nabla}^T P_{\nabla} \tilde{\nabla}}{\sigma_0^2} = \frac{\tilde{\nabla}_L^T P R \tilde{\nabla}_L}{\sigma_0^2} \tag{17}$$

where

$$\tilde{\nabla}_L = H\tilde{\nabla} \tag{18}$$

By fixing the non-centrality parameter for the alternative hypothesis based on the specification of a level of significance as well as the power of the test, the so-called minimal detectable biases can be determined. *The **internal reliability** therefore specifies the minimal biases in the observations which can be detected by means of hypothesis testing.*

*The effects of these minimal detectable biases on the final coordinate result is known as the **external reliability***. It can be computed by:

$$\nabla_X = (A^T PA)^{-1} A^T P \tilde{\nabla}_L \tag{19}$$

Here ∇_X is dependent on the coordinate definition. Baarda (1977) proposed another external reliability measure:

$$\bar{\delta}^2 = \frac{\nabla_X^T (A^T PA) \nabla_X}{\sigma_0^2} \tag{20}$$

known as the *relative* external reliability measure.

3.2 Reliability for Loop Miscloses of Simultaneous Baselines

Simultaneous baselines in the same session have almost the same design matrix (differences are at the 0.1% level). If we assume that there is a ε_i bias in the double-difference $\Delta\nabla\phi_i$ for baseline i, the biases have to be consistent with the following relation:

$$\sum_{i}^{n_b} \varepsilon_i = 0 \tag{21}$$

where n_b is the number of baselines forming a closed loop. The effect of ε_i on the coordinates has to be consistent with the relation:

$$\sum_{i=1}^{n_b} \nabla_{X_i} = \sum_{i=1}^{n_b} (A^T PA)^{-1} A^T P H \varepsilon_i = 0 \tag{22}$$

This means that the misclose from a loop of simultaneously observed baselines cannot be used to detect blunders.

As an example, if we assume all observations of satellite 13 at station 2 have a bias of 19cm, the effects on the baselines and the simultaneous loop miscloses are listed in Table 1. *The baseline vectors have been biased but the loop miscloses are unaffected.*

Table 1. The Effect of Biases at a Station on the Baseline Solution and Loop Miscloses

	No Bias	19cm Bias on Satellite 13

	x	y	z	x	y	z
Stn 1- Stn 2	-3277.4945	-2447.6874	674.6102	-3277.4049	-2447.7928	674.5341
Stn 2- Stn 3	2.3820	995.1127	-1020.1154	2.2924	995.2181	-1020.0393
Stn 3- Stn 1	3275.1123	1452.5751	345.5054	3275.1123	1452.5751	345.5054
	-0.0002	0.0004	0.0002	-0.0002	0.0004	0.0002

3.3 Reliability for Network Adjustment

The internal reliability of the network adjustment should be less than the external reliability (equation (19)) of the baseline in order to detect baseline outliers. If one GPS baseline vector i is assumed to be biased, the outlier design matrix H_i has the form:

$$H_i = \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}^T \tag{23}$$

and the variance-covariance matrix and weight matrix for this vector is C_i and P_i ($P_i = \sigma_0^2 C_i^{-1}$; σ_0^2 is the population variance factor) respectively. All baselines are assumed to be independent from the previous discussions. The internal reliability ∇_{B_i} can be derived from (equation (17)):

$$\nabla_{B_i}^T P_i R_i \nabla_{B_i} = \delta^2 \sigma_0^2 \tag{24}$$

where R_i is the ith 3×3 diagonal block in R (defined in equation (12)). The maximum and minimum internal reliabilities are in the directions of the minimum and maximum eigenvalues of $P_i R_i$. At the three axis directions, the following three internal reliabilities can be obtained:

$$\nabla_{x_i} = \frac{\sigma_0 \delta}{\sqrt{r_{x_i}}}; \qquad \nabla_{y_i} = \frac{\sigma_0 \delta}{\sqrt{r_{y_i}}}; \qquad \nabla_{z_i} = \frac{\sigma_0 \delta}{\sqrt{r_{z_i}}} \tag{25}$$

where

$$r_{x_i} = \sum_{j=1}^{3} [(P_i)_{1,j} (R_i)_{j,1}] \tag{26}$$

$$r_{y_i} = \sum_{j=1}^{3} [(P_i)_{2,j} (R_i)_{j,2}] \tag{27}$$

$$r_{z_i} = \sum_{j=1}^{3} [(P_i)_{3,j} (R_i)_{j,3}] \tag{28}$$

Note, r_{x_i}, r_{y_i} and r_{z_i} are different from the redundancy number. For correlated observations, the redundancy number cannot be a reliability measure (Wang & Chen, 1994).

The external reliabilities can be derived from equation (20):

$$\bar{\delta}_{x_i} = \delta \cdot \sqrt{\frac{P_{x_i}}{r_{x_i}} - 1}; \qquad \bar{\delta}_{y_i} = \delta \cdot \sqrt{\frac{P_{y_i}}{r_{y_i}} - 1}; \qquad \bar{\delta}_{z_i} = \delta \cdot \sqrt{\frac{P_{z_i}}{r_{z_i}} - 1} \tag{29}$$

where P_{x_i}, P_{y_i} and P_{z_i} are diagonal elements of P_i.

For a closed loop network, r_{x_i}, r_{y_i} and r_{z_i} can be simply represented as:

$$r_{x_i} = \left(\sum_{i=1}^{n_b} P_i^{-1} \right)_{1,1}^{-1}; \qquad r_{y_i} = \left(\sum_{i=1}^{n_b} P_i^{-1} \right)_{2,2}^{-1}; \qquad r_{z_i} = \left(\sum_{i=1}^{n_b} P_i^{-1} \right)_{3,3}^{-1} \tag{30}$$

where n_b is the number of baselines in the closed loop. We can see that r_{x_i}, r_{y_i} and r_{z_i} are independent of i which means all baselines in the closed loop have the same internal reliabilities. As an example, we assume a closed loop network formed by four baseline vectors derived from different sessions. The miscloses, internal reliabilities and external reliabilities are given in Tables 2 and 3 using the suggested network procedures, and using the direct output of conventional baseline processing, respectively.

Table 2. Internal and External Reliabilities for a Closed Loop Network
(Using the Proposed Procedures)

	Miscloses (cm)	Internal Reli-abilities (cm)	External Reliabilities			
			B_1	B_2	B_3	B_4
x	-0.16	3.83	3.66	11.66	11.74	10.87
y	1.59	6.06	3.50	11.46	12.26	12.33
z	-0.04	5.59	8.76	10.02	7.79	7.80

Table 3. Internal and External Reliabilities for a Closed Loop Network
(Using the Conventional Method)

	Miscloses (cm)	Internal Reli-abilities (cm)	External Reliabilities			
			B_1	B_2	B_3	B_4
x	-0.16	0.97	7.72	13.68	19.46	3.60
y	1.59	1.47	7.17	12.91	19.49	4.22
z	-0.04	1.78	18.92	15.23	19.29	3.23

From the above discussions, we make the following comments:

- The reliability measures using the direct output of conventional GPS baseline processing are too optimistic. The suggested network procedures will give more realistic reliability measures. Due to the correlation of the components of baseline vectors, the redundancy number is no longer a reliability measure.

- The simultaneous network loop has no ability to detect outliers. A site must therefore be visited twice in order to provide independent constraints.

- In order to detect baseline outliers, such as those arising from incorrect integer ambiguity resolution, before the network adjustment is performed, the number of baselines from different sessions used to form a closed loop should be justified by the internal reliability measure determined by equations (25) and (30).

4. EXPERIMENTAL RESULTS -- A MIXED MODE GPS NETWORK

5. GPS NETWORK DESIGN CONSIDERATIONS -- THE RAPID STATIC OPERATIONAL MODE

We can make use of the reliability analyses to identify network designs that can be considered the best from the network "quality" point of view.

5.1 Two GPS receivers

There are two strategies for implementing GPS rapid static positioning using two receivers. One is to operate one receiver continuously at a base site and the other receiver to move from site to site. Using this mode, a radiating star pattern is formed and each rover site should be visited at least twice. The other mode is baseline-by-baseline, or the leap-frog method. This is typical of traditional GPS static surveys. After completing the GPS rapid static survey, the network adjustment is carried out and the internal reliability should be computed. Comparing the external reliabilities (equation (19)) determined by the baseline determination procedure and the internal reliabilities determined from the network adjustment, the latter should be less than the former in order to ensure that any outliers in the baselines can be detected.

Before all GPS surveys are completed, but after a closed loop can be formed, the misclose should be used to detect outliers in the baselines of the closed loop. Internal reliabilities can indicate how many baselines are required to form a closed loop in order to detect an outlier such as that arising from wrong integer ambiguity resolution in the baseline determination procedure. If necessary further baseline observations can then be made.

5.2 Three GPS receivers

There are again two basic strategies for implementing GPS surveys using three receivers (Han et al., 1994). One is to have a receiver at a fixed base site and the other two roving from site to site. The other strategy is to have two receivers at fixed sites and one receiver roving. In an example of the first mode, in Figure 1, a receiver is fixed at station 7, the first roving receiver is set up at stations 1, 2, 3, 4, 5 and 6 and the second roving receiver is set up at stations 4, 5, 6, 1, 2 and 3. No simultaneous observations are assumed between the two roving receivers. If we assume that the travel time from one station to the next station is 20 minutes and the occupation time at each station is 10 minutes, the total survey time is 160 minutes. Six non-simultaneous closed loops (7-1-7, 7-2-7, 7-3-7, 7-4-7, 7-5-7, 7-6-7) are obtained. In the second mode, as illustrated in Figure 2, two receivers are fixed at stations 2 and 5, and the roving receiver is set up at stations 1, 6, 7, 3 and 4. The total survey time is 130 minutes and 5 simultaneous independent closed loops (2-1-5-2, 2-6-5-2, 2-7-5-2, 2-3-5-2, 2-1-4-2) are obtained. Comparing these two designs, we find that *the scheme in Figure 1 is better than that of Figure 2 from the view of reliability* because Figure 1 has six non-simultaneous closed loops formed by baselines from different sessions, while Figure 2 has five simultaneous closed loops. Although the baseline 2-5 is much more reliable, outliers occurring in stations 1, 6, 7, 3, and 4 cannot be detected successfully according to the theory in Section 3.2. The requirement for the Figure 1 design however is that the environment around station 7 should be such as to minimise any multipath effect.

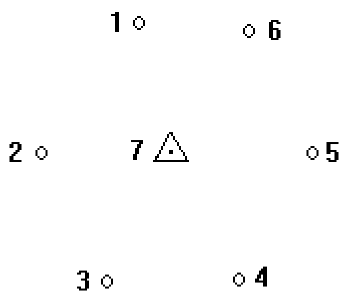

Figure 1. Fixed One Receiver Case

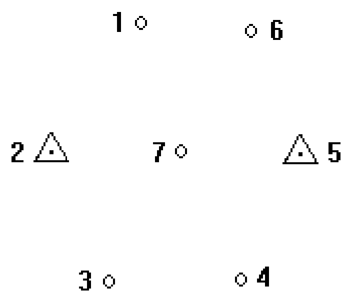

Figure 2. Fixed Two Receiver Case

5.3 Four or more GPS receivers

In the case of four receivers employed in a GPS survey, there are two reasonably good strategies of receiver deployment for the network. The first is to have one receiver at a fixed base site and the others all roving. This mode should be the best for reliability, but if the fixed receiver has some outliers this will affect three baselines. The other mode is for two receivers to be located at fixed base sites and the other two roving. In this mode, every site should be visited at least twice. In the case of more than four receivers, it is preferable that the two receivers are fixed and all other receivers are roving, and again each site should be visited at least twice.

6. CONCLUDING REMARKS

The suggested network adjustment procedures will result in better quality coordinate values, and more realistic variance-covariance matrices and reliability measures. Based on the features of network reliabilities, some receiver deployment strategies for GPS rapid static surveying are suggested.

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