

Application of 3D Time-frequency Analysis in Monitoring Full-scale Structural Response

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Abstract. Structural responses to severe loads such as due to typhoons and earthquakes are very complicated, with nonlinear and non-stationary characteristics. When external forces are applied to the structure, vibrations of different frequencies and magnitudes are excited. If any of the vibrations cannot be damped within a reasonable period of time, damage to the structure is inevitable. The damage would cause a shift of the natural frequency of the structure. Therefore it is possible to assess the integrity of the structure through monitoring this frequency shift. This has been increasingly done using integrated GPS and accelerometer systems that exploit their complementary features.

Data collected using such systems are usually analysed using the Fast Fourier Transform (FFT). The shortcoming of the FFT is that the spectrum represents the relative strength of vibrations of different frequencies over the whole data set. Recent research undertaken by the authors using Short Time Fast Fourier Transform (STFT) does give information on the temporal evolution of the dominant frequency in time series by a two-dimensional plot. However in the two-dimensional plot only the frequencies with highest amplitude are considered, other frequencies that might be of interest to the civil engineers are ignored. The significantly extended 3D spectral analysis reported in this paper can track the temporal evolution of *all* the frequency components within the detectable discrete spectrum and effectively represents the result in the three dimensions of frequency, time and magnitude. The dominant frequency can also be highlighted and tracked on the 3D mesh. The frequency domain coherent analysis based on this 3D analysis framework can further enhance common signals detected using different sensors. The 3D spectrogram analysis methodology has been applied to analyse data collected during a typhoon as well as an earthquake event with an integrated GPS and accelerometer system deployed on a 108m steel tower in Japan.

Keywords. Structural frequency response, three-dimensional spectral analysis, RTK-GPS, accelerometer, FFT, spectrum evolution.

1 Introduction

Civil structures are carefully designed based on the principles of material and structural mechanics. Various materials, and the way they are used in constructing the structures, give the structures unique dynamic characteristics such as natural frequencies, vibration modes and damping ratios, which can be predicted using Finite Element Modelling (FEM) techniques. Furthermore, wind tunnel tests on scaled models are often carried out to assist structural design (see, e.g., Penman et al, 1999). However, the quality control during construction, the maintenance during service, and the aging of the structure could alter these dynamic characteristics. Hence key man-made structures must be monitored in 'full-scale' to ensure that they maintain integrity of design, construction and operation.

Monitoring the dynamic response of civil structures for the purpose of assessment of damage has relied on measurements from inertial sensors such as accelerometers deployed on the structure. In recent years high precision GPS sensors, especially RTK-GPS systems, have been installed on large civil structures such as long span bridges, tall buildings, and on dams, side-by-side with accelerometers. In contrast, effective analysis of the data and visualise of the results from such integrated GPS and accelerometer systems has been lagged. In this paper a 3D spectral analysis framework has been proposed which can significantly improve the performance of the integrated system without increasing hardware costs.

The paper is organised as follows. The second part describes the 3D analysis framework. The third part applies the framework for the processing of data collected during an earthquake of magnitude Ms 7.0 that occurred on 26 May 2003, using an integrated GPS and accelerometer system deployed on a 108m tall steel tower. The fourth part describes the frequency domain coherent analysis and applies the methodology in case studies using both the above earthquake data and data collected during Typhoon No. 21 that struck on 1 October 2002. The paper concludes with a summary of the research findings.

2 The Framework of 3D Spectrogram Analysis

The 3D analysis framework proposed in this paper is based on the Discrete-Time Fourier Transform (DTFT) applied on a sampled signal $x[n]$ captured by sensors in the monitoring system.

The Fourier transform of a continuous signal $x(t)$ is:

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt \quad (1)$$

which is a continuous function of frequency with an amplitude of $|X(\omega)|$ (Salivahanan et al, 2000).

When $x(t)$ is sampled at time instants $t = nT$, a discrete sampled signal $x[n]$ is obtained, which is the data we process herein.

Because $\omega = 2\pi f_a$ and $T = 1/f_s$ (f_a and f_s are the analogue signal frequency and the sampling rate respectively). If given:

$$\theta = \omega T = 2\pi \frac{f_a}{f_s} \quad (2)$$

this yields DTFT of a sampled signal:

$$X(\theta) = \sum_{n=-\infty}^{+\infty} x[n] e^{-jn\theta} \quad (3)$$

Since the exponential function is periodic, the spectrum will be periodic, repeating cyclically about intervals centred on the sampling frequency. It should be noted that the frequency ranging from $\frac{f_s}{2}$ to f_s is a mirrored image of the range 0 to $\frac{f_s}{2}$ in Hz/sec (Leis, 2002). Consequently we will only take 0 to $\frac{f_s}{2}$ as the frequency response monitoring range which corresponds to the range of the digital frequency $0 \leq \theta \leq \pi$ in radians/sample.

However, the above spectrum is still in a continuous manner limited to the 0 to half sampling rate range. To calculate the DTFT an infinite number of time samples are needed from the signal (Mulgrew et al, 1999).

If we only process a finite number of data samples from $n = 0$ to $n = N - 1$ then the actual Fourier transform in Eq (3) becomes:

$$X(\theta) = \sum_{n=0}^{N-1} x[n] e^{-jn\theta} \quad (4)$$

Truncating the sampled signal into the N -sample record is equivalent to multiplying it by a rectangular window of width $\Delta t = NT$. Therefore the frequency spectrum of finite samples will be slightly distorted, because the rectangular window in the frequency domain is a sinc function.

The final step is to sample the above continuous spectrum. Only the discrete frequency points are calculated (Leis, 2002). This sampling is conventionally performed at equally-spaced points over the period extending $-\pi \leq \theta \leq \pi$. The Discrete Fourier Transform (DFT) is:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-jn(\frac{2\pi k}{N})}, k=0, 1, 2, 3, \dots N-1. \quad (5)$$

Here N is the time samples of $x[n]$, and the number of frequency samples k equals to N . Consequently, the frequency resolution or the frequency interval between the DFT values is $\Delta f = \frac{f_s}{N}$. This “windowing” process

introduces a slight distortion into the frequency representation of the signal being analysed but it can be improved by applying a window function, e.g. Hamming window (Goswami & Chan, 1999). Furthermore, due to the conjugate symmetry of the DFT, half of the output is redundant. The frequency samples will only be taken up to $\frac{N}{2} - 1$.

Thus for a signal with $N \times M$ samples, tracking its power spectrum as it changes over time would be possible, and allows us to monitor a structural natural frequency vibration induced by strong winds or earthquakes. In order to detect and track the response signatures “instantaneously” in 3D, select a suitable sample number N in power of two for the FFT to generate the DFT spectrum for a particular time instant. Therefore, we can have M blocks of the DFT spectrum in the short $\Delta t = NT$ duration, each of length $\frac{N}{2}$.

localised spectrums thus contain the time information. The time measurement in one segment length is $t = \Delta t \cdot M$ with a resolution of $\Delta t = NT$, and the frequency ranging from 0 to half sampling rate with a resolution of $\Delta f = \frac{f_s}{N}$. Therefore, the discrete time

$t[m] = (1 : M) \Delta t$ and the discrete frequency $F[n] = (0 : \frac{N}{2} - 1) \Delta f$ form a mesh function. Mapping

the magnitude $|X[k]|$ at discrete frequencies onto the 2D time and frequency mesh, the 3D time-frequency-strength analysis is performed. This can reveal the frequency composition of the signal at each time step so that the occurrence of transients can be pinpointed.

When N is selected small enough the time interval will be close to zero and, hence, the DFT can be considered as an instantaneous output. Therefore, the waveform evolution (relative strength) of frequency composition can be observed with respect to both time

and frequency. The difference herein with traditional power spectral density computation is that we do not square the amplitude at each frequency by the number of data samples (Mulgrew et al, 1999). There is direct interest in correlating measured dynamic properties of natural frequency and damping to specific levels of response amplitude in full-scale structural monitoring. The magnitude spectrum will contain a peak at the corresponding frequency. Tracking the peaks over time allows us to find the frequency of interest and its duration, and compare with the original designed structural free natural frequency modes.

3 Case Studies

An integrated RTK-GPS and accelerometer system for full-scale structural deformation monitoring has been deployed on a steel tower in Tokyo of 108m in height, together with other sensors such as anemometer and strain gauge. The RTK-GPS and accelerometer data were recorded at 10 and 20Hz sampling rates respectively. The above proposed three-dimensional spectrogram analysis framework has been applied to data collected from 18:00 to 19:00 Japan Standard Time when an earthquake event of magnitude Ms 7.0 occurred on 26 May 2003 at a depth of 71km (Li, 2004).

3.1 Traditional FFT analysis

The overall plots of time series of the RTK-GPS measured displacements in X, Y and Z directions are shown in Figure 1. From the least-squares polynomial fitting (blue lines) in this figure the maximum displacements in the X and Y directions are all less than +/-1cm during the two hour period, indicating no significant static or quasi-static movements. But there is the peak-to-peak displacements of around 6cm caused by seismic waves in the two directions after the 1500sec mark. While in the Z direction after the 1500sec mark, the polynomial fitting result is more than +/-2cm.

The traditional Fast Fourier Transform (FFT) was used to detect the structural response induced by this quake from the RTK-GPS measurements (Figure 2). Studying the three spectrums closely by zooming in, it is obvious that the largest amplitude at 0Hz presents a strong static movement. And semi-static movements are contributing to all three components, centred around the frequency of 0.15Hz, this could be due to GPS multipath effects as well. In the mean time, both in the X and Y directions there are peaks at 0.57Hz and 2.16Hz. There is no clear peak in the Z direction at the expected signals, but signal or noise at the lower frequency end (0-0.8Hz) is much stronger than in the other two directions. This can be partly explained as the semi-static movements

during the quake affecting the tower mostly along the Z direction. This agrees with the time series shown in Figure 1.

Measurements from the accelerometers (X and Y directions only) are given in Figure 3. Note the acceleration (Figure 3) has been limited to a period of 300 seconds in order to see both the P and S waves clearly. The peak-to-peak acceleration is almost 200cm/s^2 .

The accelerometer's performance during the seismic event was analysed using FFT as well (Figure 4). There are three peaks all in the same locations in the X and Y directions, of which the 0.57Hz and 2.16Hz peaks are identical to the GPS spectrum, while the 4.58Hz is a high order harmonic that is not seen in the GPS spectrum. The very clear difference compared to the GPS spectrum is that these spectrums do not contain static and semi-static components at all.

However, the FFT spectrums give only a "global" picture of the frequency compositions, i.e. over the whole time series. But the quake-induced signal is not stationary. How can we obtain the time varying characteristics of the frequency compositions?

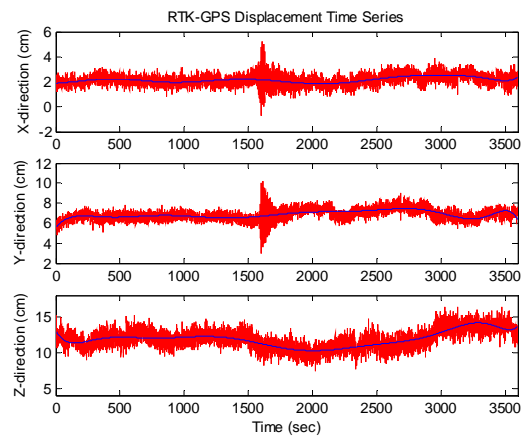


Figure 1 RTK-GPS measured displacements.

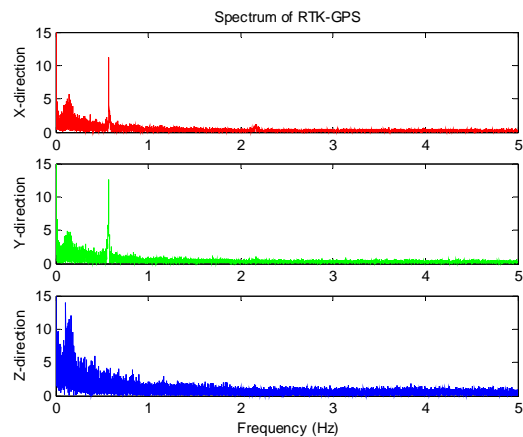


Figure 2 FFT spectrums of the GPS.

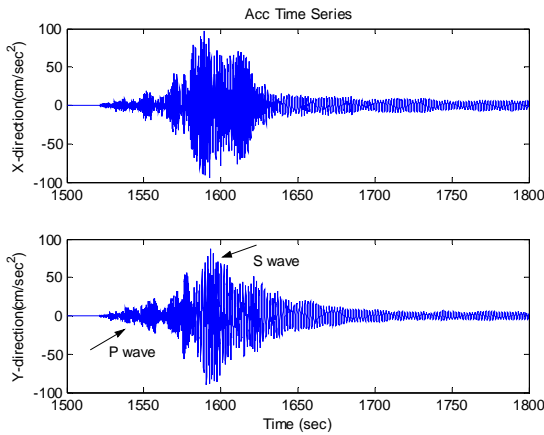


Figure 3 Acceleration time series measured by accelerometers.

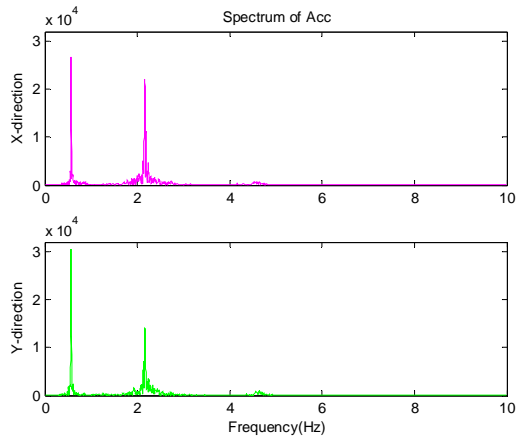


Figure 4 FFT spectrum of the acceleration.

3.2 Two Dimensional Time-frequency Analysis Using Short Time Fourier Transform

In order to overcome the shortcomings of the traditional FFT analysis so that the time-varying features of the structural response can be revealed, we adopt the concept of Short Time Fourier Transform (STFT) from speech signal processing. By extracting the maximum peak in the “instantaneous” frequency spectrum and tracking its evolution with time, the newly formed 2D time-frequency spectrum plot would give us information on when there is a dominant frequency shift, i.e. indicate how dominant frequencies evolve with time (Li et al, 2004).

Figure 5 is the 2D plot for the GPS measurements in the Y direction showing the abrupt frequency shift clearly in the time series. It can be seen that the tower experienced the strongest vibration lasting for about 7 minutes around 18:30 when the tower’s first order natural frequency 0.57Hz became dominant. This cannot be determined from the GPS displacement time series

itself because it is too noisy (cf. Figure 1). Nor can it be determined from the global FFT spectrum because of the lack of time information. It does show that before and after the quake the dominant frequencies are centred around 0.15Hz, which agrees with the global FFT result.

It is interesting to see that this duration estimated from the accelerometer measurements in the Y direction is about 14 minutes, as shown in Figure 6. Note that the starting time for the 0.57Hz signal is the same for both the GPS and accelerometer. The finishing time is different because the amplitude of seismic waves decreases gradually and when the induced displacement is smaller than the RTK-GPS noise floor it cannot be tracked by GPS anymore. The seismic-induced acceleration, however, will continue to be tracked by the accelerometer for as long as the 0.57Hz frequency is dominant. Figure 6 does show that before and after the quake the dominant frequencies are centred around 2.16Hz, which agrees with the global FFT result.

Therefore the STFT is a significant step forward from the traditional FFT in terms of structural response analysis. But we have focused on only the dominant frequency. What about the other frequency compositions? In fact any induced frequency mode is very useful in monitoring the damping characteristics of the structure (Tamura, 2003). Thus, it is crucial to develop a three-dimensional spectrogram framework which retains all the spectrum analysis information.

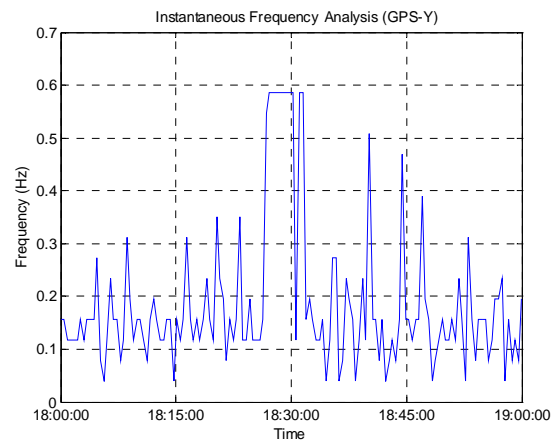


Figure 5 Frequency evolution on GPS position (y-component).

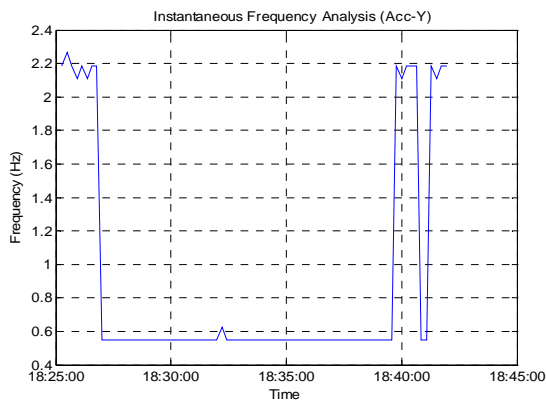


Figure 6 Frequency evolution on acceleration (y-component).

3.3 Three Dimensional Time-frequency Spectral Analysis

The 3D spectrogram framework outlined in section 2 has been designed to address one of the 2D spectrum analysis major shortcomings: focusing only on dominant frequencies. The 3D framework can reveal not only the timing but also the relative strength of the dominant frequencies. As a matter of fact, it gives the temporal evolution of the full frequency spectrum. Figure 7 is an example of such a 3D analysis applied to the acceleration (Y component). Three modes of induced frequency response are visible, each with a certain bandwidth. They have been confirmed by FEM analysis. The pink line tracks the dominant frequencies. If we ignore all the other frequencies and project this pink line onto the time-frequency plane, it will give the same result as Figure 6.

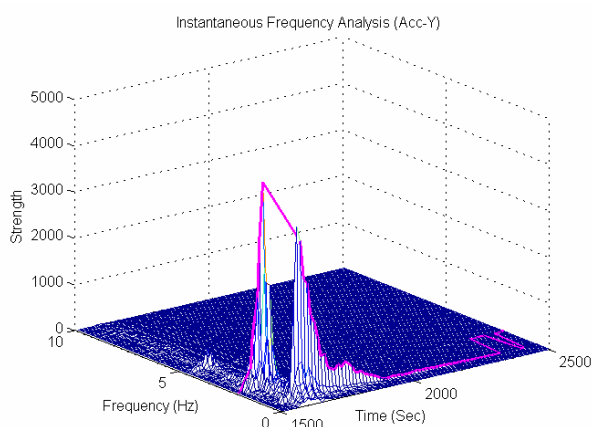


Figure 7 The 3D spectrogram: temporal evolution of the full frequency spectrum.

However, this 3D result gives much richer information. For example, zooming-in on the 1500-2000 second portion of Figure 7, Figure 9 shows clearly how

the quake-induced response around the 1st mode natural frequency 0.57Hz decayed quickly, so that the quake-excited vibration did not cause damage to the structure, which vividly proved the effectiveness of the damping design of the steel tower.

The 3D analysis is applied on the GPS measurements as well (Figure 8). In addition to the static, semi-static, and 1st mode (0.57Hz) signals, the 2nd mode (2.16Hz) can be seen much more easily around the time of the quake, which was not so obvious in the 2D analysis.

Comparing Figures 8 and 9, it can be seen that when the quake occurred the dominant frequency of the GPS measurements transits from static/semi-static to the 1st mode but returns to static/semi-static after the event while the acceleration moves from the 2nd mode to the 1st mode and then returns to 2nd mode. Therefore, the two sensors are complementary in terms of monitoring capability.

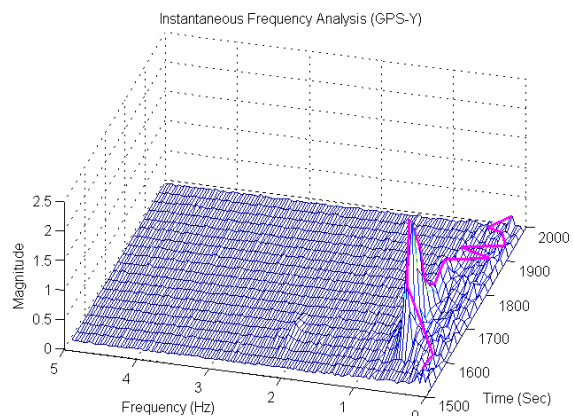


Figure 8 3D spectrogram of GPS position (Y component).

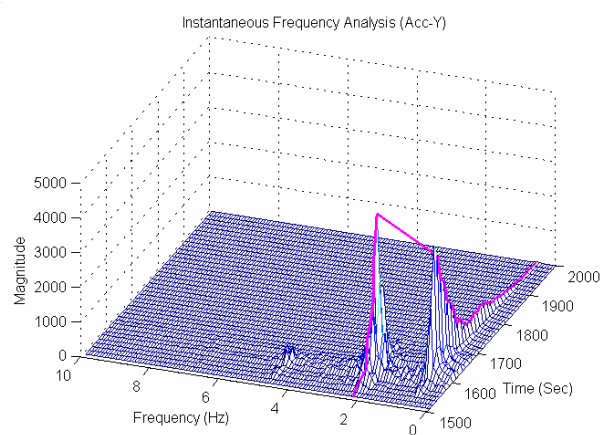


Figure 9 3D spectrogram of acceleration (Y component).

4 Frequency Domain Coherent Analysis

Despite the obvious complementarity between the RTK-GPS and accelerometer, so far the observations have been analysed separately, be it in 2D or 3D. Similar to laser interferometry, it is possible to obtain a coherent frequency spectrum by multiplying the structural frequency responses measured by the two different sensors.

The advantage of this frequency domain coherent analysis is that the common strong signals detected by both sensors could be further enhanced, while the sensor specific noise would be suppressed. In a sense, the 3D frequency response obtained from one sensor can be used as a filter for that of the other sensor. If Sensor 1 has the frequency response $X_1[k]$ and Sensor 2's is $X_2[k]$, then the coherent frequency response $Y[k]$ is given by:

$$Y[k] = X_1[k] \bullet X_2[k]$$

However, there is a challenge when applying the coherent analysis on data collected using the system deployed in Tokyo. In this system we use a 20Hz sampling rate for the accelerometer and 10Hz for RTK-GPS. Therefore, the spectrum is 0 to 10Hz for acceleration and 0 up to only 5Hz for displacement. If we keep the same frequency resolution for both GPS and accelerometer measurements, the window length for acceleration will be twice that of the RTK-GPS displacement. As a result, the 3D acceleration spectrum has to be truncated by half so that the derived acceleration spectrum has the same size as the RTK-GPS.

This proposed methodology has been applied to two sets of data: one was collected from 18:00 to 19:00 Japan Standard Time when an earthquake event of magnitude Ms 7.0 occurred on 26 May 2003; and another collected when Typhoon No. 21 struck Tokyo on 1 October 2002.

4.1 Coherent analysis between GPS position and acceleration (earthquake)

The spectrums of RTK-GPS and accelerometer are given in Figures 8 and 9. Figure 10 shows the result of coherent analysis in the frequency domain. It can be seen that the 1st and 2nd modes have been enhanced and their bandwidths narrowed. In fact, it is a result of integration of the two sensors in frequency domain. The very smooth background has shown us the effectiveness of coherent analysis.

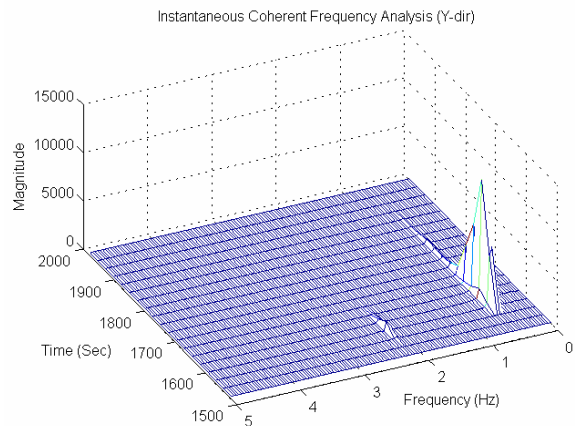


Figure 10 Result of coherent analysis between GPS position and acceleration (earthquake event).

4.2 Coherent analysis between GPS position and acceleration (typhoon)

Figures 11-13 show the coherent analysis between GPS and accelerometer (X component). Figure 11 shows that the accelerometer picked up mainly the 1st mode free vibration of the structure under the typhoon load. RTK-GPS, on the other hand, picked up both this 1st mode and the static and semi-static movements of the tower, the latter being more significant. The coherent analysis revealed the overlap between the two spectrums nicely, i.e. the 1st mode, in Figure 13. Comparing this and Figure 10 it can be concluded that the quake was more powerful than the typhoon in terms of dynamic force so that not only the 1st but also the 2nd mode vibration was excited. On the other hand, strong winds could cause a continuous static and semi-static movement. Therefore it would be useful to study wind speed and its induced displacement and acceleration, which will be published in a separate paper.

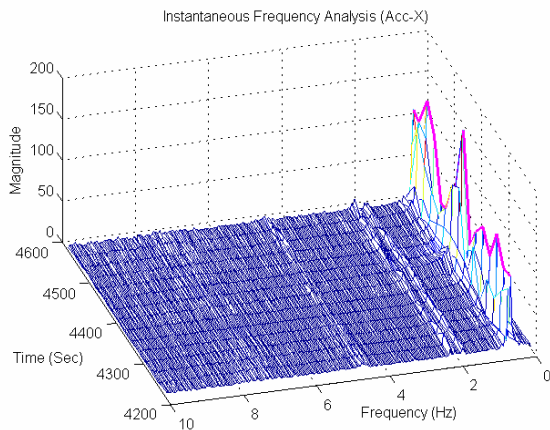


Figure 11 The 3D spectrogram of acceleration (typhoon event).

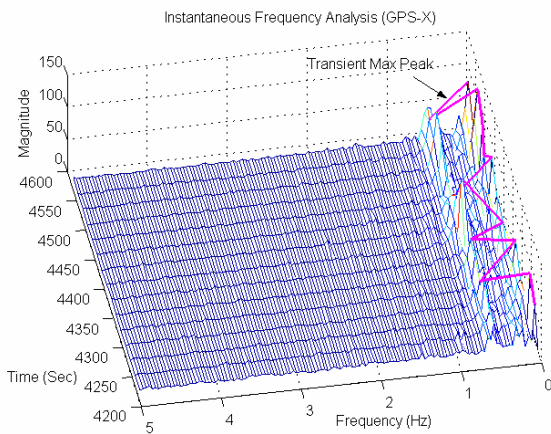


Figure 12 The 3D spectrogram of GPS position (typhoon event).

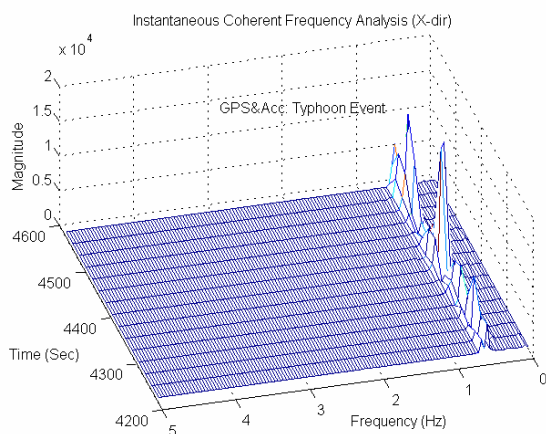


Figure 13 Result of coherent analysis: GPS and accelerometer (typhoon event).

5 Concluding Remarks

Data collected using combined RTK-GPS and accelerometer systems are usually analysed with the traditional Fast Fourier Transform (FFT) method, and separately for each sensors. The shortcoming of this approach is that the spectrum represents the relative strength of vibrations of different frequencies over the whole data set. It is a *frequency-magnitude two-dimensional* analysis and the complementarity between the two sensors is not exploited.

Recent research undertaken by the authors using short time Fast Fourier Transform (STFT) does give information on the temporal evolution of the dominant frequency, which represents a *time-dominant frequency two-dimensional* analysis. The shortcoming of the method is that the focus is only one or a few major frequency components, but other frequencies might be of interest also.

The significantly extended three-dimensional analysis reported in this paper can track the temporal evolution of *all* the frequency components and effectively represents the result in the *3D of frequency, time and magnitude*. The dominant frequency can also be tracked on the 3D mesh.

The frequency domain coherent analysis based on this 3D analysis framework can further *enhance common signals between sensors*.

This 3D analysis methodology has been applied to data collected during a typhoon as well as an earthquake event with an integrated GPS and accelerometer system deployed on a 108m tall steel tower in Japan. The proposed framework can significantly improve the visualized performance of the integrated system without increasing hardware costs.

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