

# Differential Early Late Phase for Multipath Detection at Critical Doppler Offsets

Omer Mohsin Mubarak, *The University of New South Wales, Sydney Australia*  
Andrew G Dempster, *The University of New South Wales, Sydney Australia*  
[omer@student.unsw.edu.au](mailto:omer@student.unsw.edu.au)

## BIOGRAPHY

Omer Mohsin Mubarak is a PhD student in the School of Surveying and Spatial Information Systems at the University of New South Wales, Australia. He completed a Master of Engineering in Electrical Engineering from the same university in 2006 and a Bachelor of Science in Electronics Engineering from Ghulam Ishaq Khan Institute of Engineering Sciences and Technology, Pakistan in 2002. His current research is focused on multipath mitigation using modernized GNSS signals.

Andrew Dempster is Director of Research in the School of Surveying and Spatial Information Systems at the University of New South Wales. He was project manager and system engineer on the first GPS receiver to be designed in Australia and has been teaching satellite navigation for over a decade. His current research interests are in signal processing for satellite navigation, new receiver techniques and new location technologies.

## ABSTRACT

Early late phase (ELP) is a difference in phase of early and late correlator outputs. It is higher in the presence of multipath, which makes it effective for detecting the presence of multipath. It has been found that at certain Doppler offsets, averaged ELP is higher even in the absence of multipath [2]. This can cause false alarms for multipath detection. This paper presents a differential ELP (DELPE). It has been shown that it has a lower value at these critical carrier frequencies of the satellite signal in a given channel. These critical carrier frequencies can be divided into two types, fixed critical frequencies and variable critical frequencies.

On the basis of mathematical models for ELP [2], it is proposed to locally generate the estimate of ELP when the IF of a desired satellite is within 50 Hz of any critical frequency. Around fixed critical frequencies, an estimate of ELP is generated using the tracking loop outputs of the desired signal channel for the given integration period.

Around variable critical frequencies, an ELP estimate is generated using the tracking loop outputs for the desired signal channel and an interfering one. In most receivers, it is likely that the integration period of different channels would not be synchronized with each other in time. Thus, in this case the tracking loop outputs from interfering channels are interpolated for the time duration of integration period of the current channel.

These estimates for ELP are then subtracted from the ELP of the received signal to obtain DELPE. As DELPE is significantly lower than ELP at critical frequencies, a lower threshold can be set for multipath detection for DELPE as compared to ELP. This is helpful for detecting multipath when the reflected signal is relatively weaker when compared to the LOS signal or the phase difference between the two is close to integer multiples of  $\pi$ . This paper also analyzes DELPE in the presence of multipath in the satellite signal being tracked as well as in the interfering one.

## INTRODUCTION

Multipath is one of the major sources of error in navigation systems, especially in indoor and urban environments. In urban environments, it is possible that a reflected signal is received from the satellite in addition to or instead of the line of sight (LOS) signal. The reception of more than one signal from the same transmitter is known as multipath. The reflected signal is always received at the receiver with some time delay compared to the LOS signal due to its longer path. It may be stronger or weaker than the LOS signal. In the presence of both LOS and reflected signals, the two signals have different carrier phases when they reach the receiver because of the time delay between them. Recently, this difference has been exploited for effective multipath detection. A novel parameter, early late phase (ELP) has been proposed. It is calculated as the difference in phase of early and late correlator outputs [1]. As opposed to most other methods for multipath mitigation which use the magnitude of correlator outputs, ELP is calculated from their phase.

ELP is averaged in time to reduce thermal noise effects. It has been found that at certain Doppler offsets, averaged ELP is higher even in absence of multipath [2]. This can cause false alarms for multipath detection. This paper presents a differential ELP (DELP), which has lower value at these critical carrier frequencies of the satellite signal in a given channel.

The paper first presents a brief introduction to early late phase and then identifies the critical Doppler offsets. Then DELP is described and results are presented showing that it is better than ELP for avoiding false alarms in multipath detection. Lastly, DELP is analyzed for the multipath environments. Results presented in this paper are obtained using a software receiver [4].

### EARLY LATE PHASE

In the presence of multipath, the carrier tracking loop in the receiver is locked to a phase which is somewhere between the phase of the LOS signal and that of the reflected one [1]. The phase error between the LOS signal and local signal depends on the relative amplitude of the reflected signal with respect to the LOS one and the phase difference between the two. The carrier tracking loop adjusts the phase to keep maximum energy in the I channel of the prompt correlator. This means that the phase of the prompt correlator always remains close to zero. However, the presence of the same signal at two carrier phases results in nonzero phases of early and late correlators. The phase of one of these moves toward positive and another one towards negative in the presence of multipath. Thus, ELP, which is the difference of the two, is higher in the presence of multipath and shown to be a useful discriminator to detect multipath's presence [3].

### CRITICAL DOPPLER OFFSETS

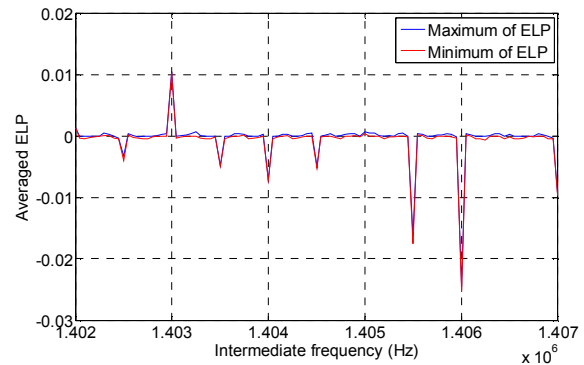
ELP is averaged in time to reduce the thermal noise effects. It has been found that at certain Doppler offsets, averaged ELP is higher even in absence of multipath [2]. This can cause false alarms for multipath detection, as an increase in ELP magnitude otherwise corresponds to the presence of multipath. In this paper, these Doppler offsets are termed as critical Doppler offsets. They can be divided into two types, fixed critical Doppler offsets and variable critical Doppler offsets.

#### *Fixed critical Doppler offsets:*

All the correlator outputs in a receiver have a residual carrier and as ELP is calculated from them, it has the same frequency component as well. In the absence of multipath, this carrier in the ELP has a mean of zero and

thus averaged ELP is zero. However, whenever the intermediate frequency (IF) is an integer multiple of 500 Hz, the residual carrier becomes a DC value and as a result ELP is averaged to a nonzero value [2].

Figure 1 shows the averaged ELP for PRN 12 in a noiseless environment with an integration period of 1 msec [2]. It can be seen that the averaged ELP has higher magnitude at every IF of 500 Hz integer multiples. The Doppler offsets in the satellite carrier frequency which cause such an IF are termed “fixed critical Doppler offsets”. Fixed critical Doppler offsets are known a priori and remain same for the given receiver. The increase in ELP at these offsets is dependent on the Doppler offset in the carrier frequency and PRN code for the desired satellite.



**Figure 1 – ELP averaged over 20 msec in absence of multipath (PRN 12) [2]**

#### *Variable critical Doppler offsets:*

Figure 2 shows the averaged ELP for PRN 12 in the presence of satellite PRN 25 with IF of 1405150 Hz [2]. It can be seen that the ELP magnitude is higher at each IF whose difference or sum with IF of PRN 12 is equal to an integer multiple of 1 kHz, in addition to fixed critical Doppler offsets. These additional Doppler offsets are termed “variable critical Doppler offsets”. Variable critical frequencies depend on the Doppler offset of carriers of other interfering satellite signals present and thus they change over time. The increase in ELP in this case is dependant on carrier frequency difference between the two satellites, their PRN codes and the phase offset between their PRN codes.

### DIFFERENTIAL EARLY LATE PHASE

This paper presents a differential ELP (DELP), which has a lower value at the critical Doppler offsets of the carrier frequency of the satellite signal as compared to ELP. Moreover, in the presence of a given multipath, the numerical value of DELP is approximately equal to that of ELP. As DELP evaluation requires more processing than that of ELP, it is proposed that DELP should be used instead of ELP only around critical Doppler offsets. For

ELP averaging time of 20 msec, DELP is used instead of ELP within  $(20\text{msec})^{-1} = 50$  Hz of critical Doppler offsets. This ensures that whenever ELP is used for multipath detection; there is at least one complete cycle of residual carrier in an averaging period.

DELP is calculated as the difference of ELP of the incoming signal and the locally generated ELP for a given set of conditions. These conditions include code and carrier information of the incoming signal, which can be obtained from tracking loop outputs.

### DELP at fixed critical Doppler offsets:

An estimate of the output of the late correlator in the Q-channel can be given by equation 1 [2].

$$Q_{Le}(x) = \sum_{k=0}^{s-1} \begin{bmatrix} \sin\left(\frac{2\pi f}{FS}(x+k)\right) \\ \cos\left(\frac{2\pi f}{FS}(x+k)+\theta\right)c(x+k)c(x+k-\varepsilon) \end{bmatrix} \quad (1)$$

where subscript  $e$  corresponds to estimate,  $f$  is the IF,  $FS$  is the sampling frequency,  $c$  is the PRN code of the satellite being tracked,  $s$  is the number of samples per integration interval,  $\varepsilon$  is the correlator spacing and  $\theta$  is the carrier phase error, which should ideally be equal to zero. Similarly, other correlator outputs can also be computed. Thus, an estimate of ELP for a given satellite signal in absence of multipath, noise and cross correlation can be given by equation 2.

$$ELP_e(x) = \tan^{-1} \left( \frac{\sum_{k=0}^{s-1} \begin{bmatrix} \cos\left(\frac{2\pi f}{FS}(x+k)\right) \\ \sin\left(\frac{2\pi f}{FS}(x+k)+\theta\right)c(x+k)c(x+k+\varepsilon) \end{bmatrix}}{\sum_{k=0}^{s-1} \begin{bmatrix} \sin\left(\frac{2\pi f}{FS}(x+k)\right) \\ \sin\left(\frac{2\pi f}{FS}(x+k)+\theta\right)c(x+k)c(x+k+\varepsilon) \end{bmatrix}} \right)$$

$$- \tan^{-1} \left( \frac{\sum_{k=0}^{s-1} \begin{bmatrix} \cos\left(\frac{2\pi f}{FS}(x+k)\right) \\ \sin\left(\frac{2\pi f}{FS}(x+k)+\theta\right)c(x+k)c(x+k-\varepsilon) \end{bmatrix}}{\sum_{k=0}^{s-1} \begin{bmatrix} \sin\left(\frac{2\pi f}{FS}(x+k)\right) \\ \sin\left(\frac{2\pi f}{FS}(x+k)+\theta\right)c(x+k)c(x+k-\varepsilon) \end{bmatrix}} \right) \quad (2)$$

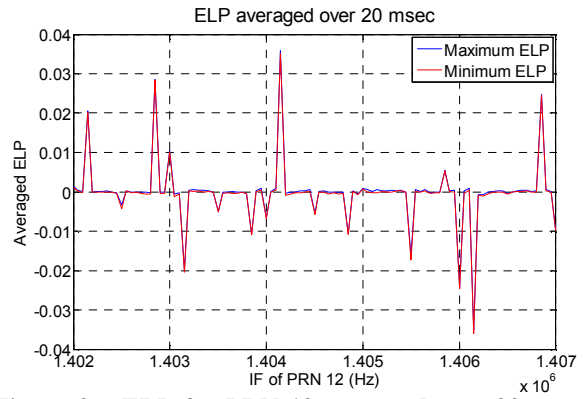


Figure 2 – ELP for PRN 12 averaged over 20 msec in presence of PRN 25 (IF=1405150 Hz) and absence of multipath [2].

In order to calculate estimated ELP, parameters on the right hand side of the equation 2 are obtained from the tracking loop output.  $f$  is obtained directly from carrier tracking loop,  $s$  is calculated using the code frequency from code tracking loop,  $\varepsilon$  and  $FS$  are fixed for a given receiver and  $\theta$  is assumed to be zero. The ELP of the received signal can be calculated using correlator outputs, as given in equation 3.

$$ELP_c(x) = \tan^{-1} \left( \frac{Q_{Ec}(x)}{I_{Ec}(x)} \right) - \tan^{-1} \left( \frac{Q_{Lc}(x)}{I_{Lc}(x)} \right) \quad (3)$$

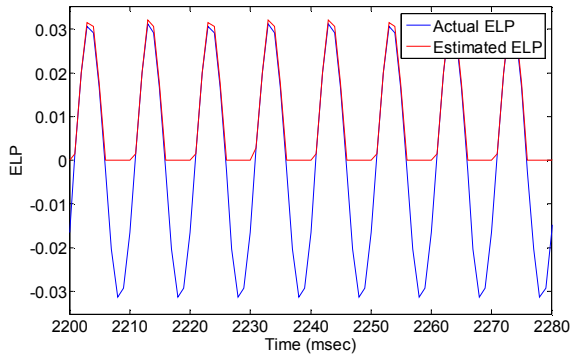
where subscript  $c$  corresponds to the value calculated directly from the received signal. DELP is defined as the difference between the estimated and calculated ELP, given by equation 4.

$$DELP(x) = ELP_c(x) - ELP_e(x) \quad (4)$$

DELP is also averaged in time to reduce the effect of thermal noise.

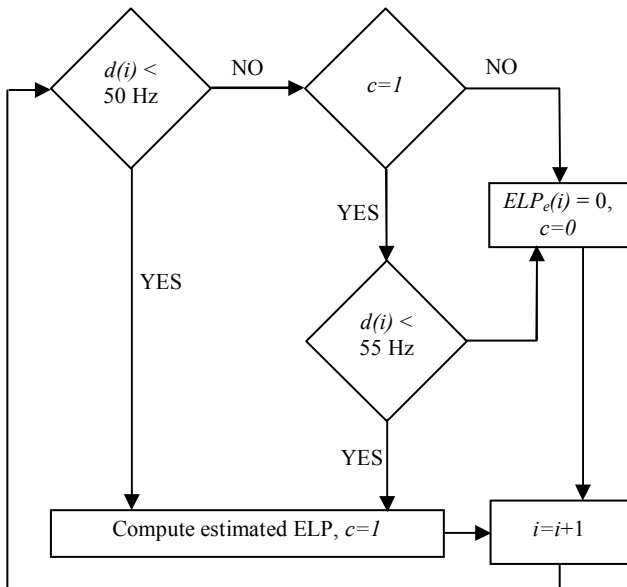
As mentioned before, DELP is used instead of ELP within 50 Hz of fixed critical Doppler offsets. This implies that the carrier frequency estimated by the tracking loop will be checked continuously and estimated ELP is computed when it is within 50 Hz of any critical Doppler offset. However, the carrier frequency given by the carrier tracking loop also has residual carrier, which is present in all correlator outputs [2]. This residual carrier can cause a problem at the 50 Hz boundaries around critical Doppler offsets. Figure 3 shows the estimated and mathematically calculated ELP at IF of 1405050 Hz, which is exactly 50 Hz away from the critical IF of 1.405 MHz. It can be seen that since the IF is checked for every integration period, the estimated ELP is only calculated for half of the cycle, when the numerical value of carrier frequency given by the tracking loop is within 50 Hz. It can cause averaged

DELP to have higher magnitude and thus can result in a false alarm for multipath detection.



**Figure 3 – Actual and estimated ELP for PRN 12 for IF of 1405050 Hz in absence of multipath**

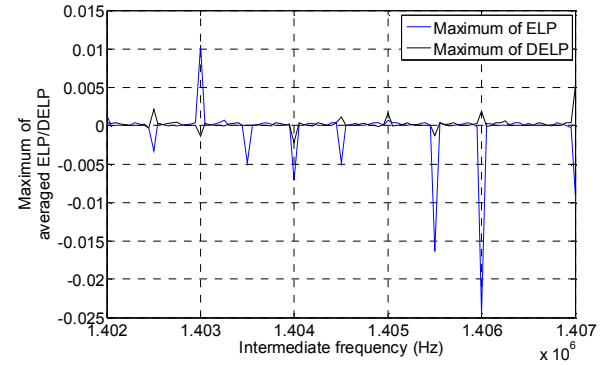
Thus, a simple algorithm shown in Figure 4 is proposed to determine if estimated ELP needs to be calculated or not. It simply increases the boundary around critical Doppler offsets from 50 Hz to 55 Hz, if ELP was estimated for the previous integration period. It will ensure that ELP is not estimated for half of the cycle if the frequency output from the carrier tracking loop goes slightly beyond the 50 Hz boundary due to residual carrier or thermal noise. In the Figure 4,  $d(i)$  is the absolute difference between the IF of the satellite being tracked and the IF for the nearest critical Doppler offset for the  $i^{th}$  integration period.  $c$  is the flag to check if ELP was estimated for the previous integration period.



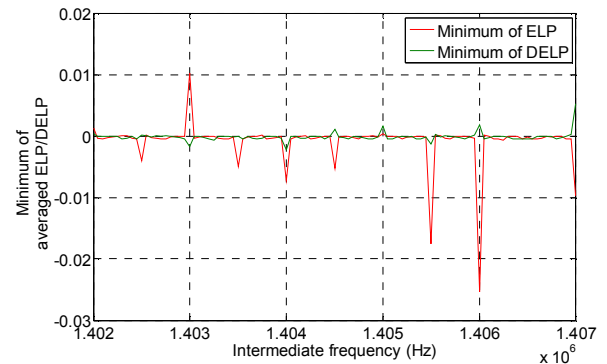
**Figure 4 – Algorithm to determine whether to calculate estimated ELP or not**

Figure 5 and Figure 6 show the plots for the maximum and minimum of averaged ELP and DELP. It can be seen that in both cases the magnitude of DELP is much lower

than that of ELP at fixed critical Doppler offsets. Thus, a lower threshold can be set for DELP as compared to ELP to detect multipath. A lower threshold is beneficial for detecting weak reflections and the ones which have relative phase offset with the LOS signal close to integer multiples of  $\pi$ .



**Figure 5 – Comparison of maximum values of averaged ELP with that of DELP in a noiseless environment**



**Figure 6 – Comparison of minimum values of averaged ELP with that of DELP in a noiseless environment**

**DELP at variable critical Doppler offsets:**

Similar to fixed critical Doppler offset, DELP can also be used around variable critical Doppler offsets. However, estimation of ELP around variable critical Doppler offsets is more complicated than around fixed ones. In this case, code and carrier information of both the tracked and interfering satellite are required. Moreover, if the integration periods of the two channels are not synchronized in time, the phase information of the interfering satellite needs to be interpolated for the integration period of the satellite being tracked.

In order to estimate ELP, first let us estimate the received signal. The received signal of a satellite in the presence of another satellite in an otherwise noiseless environment can be given by equation 5.

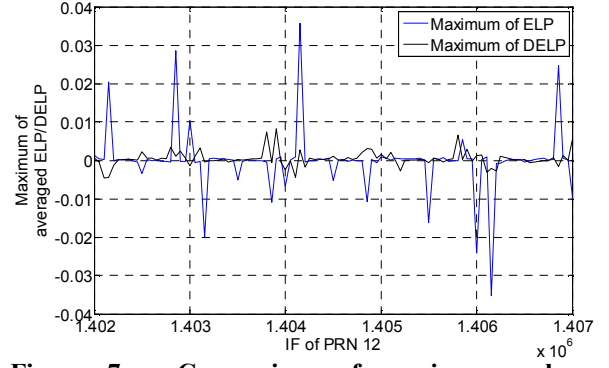
$$\begin{aligned}
R_e(x) &= \sin\left(\frac{2\pi f_1}{FS}x + \theta_1\right)c_1(x) + \\
&\sin\left(\frac{2\pi f_2}{FS}x + \theta_2 + 2\pi f_2 L\right)c_2(x + Lf_o)
\end{aligned} \tag{5}$$

where subscript 1 corresponds to the satellite being tracked and 2 corresponds to the interfering one. L is the time difference between the start of integration period for the two signals and  $f_o$  is the code frequency of the interfering satellite. Thus, the estimate of ELP can be given by equation 6.

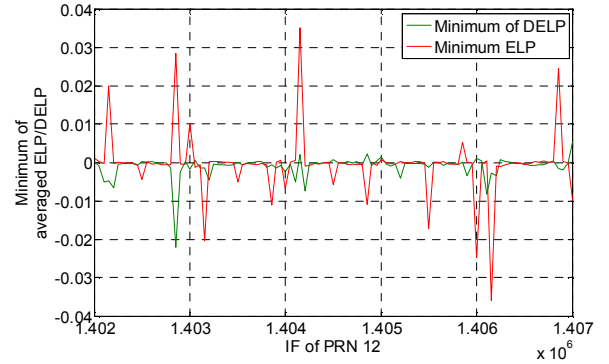
$$\begin{aligned}
ELP_e(x) &= \tan^{-1} \left[ \frac{\sum_{k=0}^{s-1} \left[ \begin{array}{l} R_e(x+k) \cos\left(\frac{2\pi f}{FS}(x+k)\right) \\ c_1(x+k+\varepsilon) \end{array} \right]}{\sum_{k=0}^{s-1} \left[ \begin{array}{l} R_e(x+k) \sin\left(\frac{2\pi f}{FS}(x+k)\right) \\ c_1(x+k+\varepsilon) \end{array} \right]} \right] \\
&- \tan^{-1} \left[ \frac{\sum_{k=0}^{s-1} \left[ \begin{array}{l} R_e(x+k) \cos\left(\frac{2\pi f}{FS}(x+k)\right) \\ c_1(x+k-\varepsilon) \end{array} \right]}{\sum_{k=0}^{s-1} \left[ \begin{array}{l} R_e(x+k) \sin\left(\frac{2\pi f}{FS}(x+k)\right) \\ c_1(x+k-\varepsilon) \end{array} \right]} \right]
\end{aligned} \tag{6}$$

Similar to the estimation of ELP at fixed critical Doppler offsets, the estimate of parameters on the right hand side of the equation 6 can also be obtained from the code and carrier tracking loop outputs. Then equation 3 can be used to obtain the ELP of the received signal and equation 4 can be used to calculate DELP. Also, DELP can be used instead of ELP within 50 Hz of variable critical Doppler offsets and the algorithm in Figure 4 can be used to determine if estimated ELP needs to be calculated or not.

Figure 7 and Figure 8 show the plots for the maximum and minimum of averaged ELP and DELP for PRN 12 in the presence of PRN 25 in an otherwise noiseless environment. It can be seen that in both cases the magnitude of DELP is much lower than that of ELP at variable critical Doppler offsets. As ELP estimates at critical Doppler offsets require an estimate of the phase of the signal from the satellite being tracked as well as from the interfering one, an error in any one of the tracking loop outputs can cause an erroneous estimate. Not accounting for this, it can be seen that in general the magnitude of DELP at fixed critical Doppler offsets is lower than that at variable ones.



**Figure 7 – Comparison of maximum values of averaged ELP with that of DELP for PRN 12 in the presence of PRN 25 (IF=1405150 Hz)**



**Figure 8 – Comparison of minimum values of averaged ELP with that of DELP for PRN 12 in the presence of PRN 25 (IF=1405150 Hz)**

## DELP – IN THE PRESENCE OF MULTIPATH

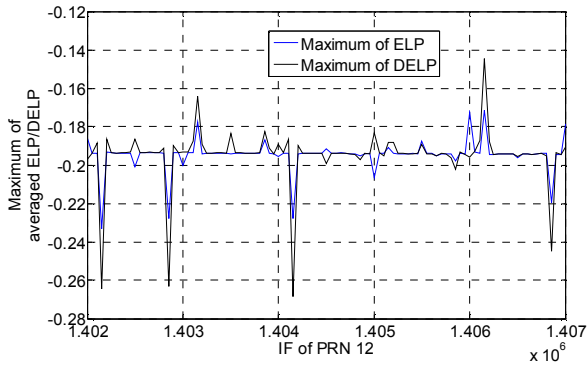
In the previous section, it has been found that DELP is better than ELP because it is less sensitive to critical Doppler offsets in absence of multipath and thus has lower probability of false alarm. This section analyses DELP in the presence of multipath in the signal of satellite being tracked as well as in the interfering satellite signal. The results are then compared with those obtained using ELP.

### *Multipath in the satellite signal being tracked:*

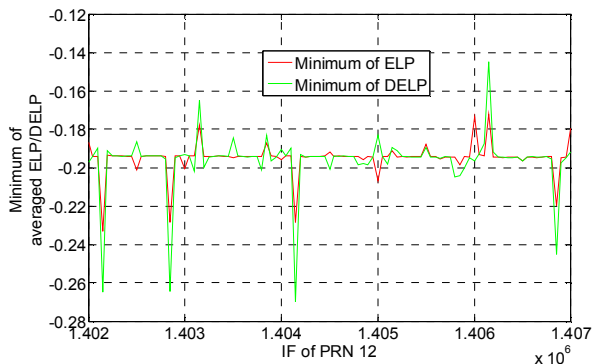
Figure 9 and Figure 10 show the plots for the maximum and minimum of averaged ELP and DELP for PRN 12 in the presence of PRN 25 with multipath added to the PRN 12 signal. It can be seen that the increase in ELP due to multipath is significantly higher than the one because of residual carrier at critical Doppler offsets. However, the ELP value in the presence of multipath is dependant on the relative magnitude and phase of the reflected signal. Whenever the amplitude is lower or the phase is close to an integer multiple of  $\pi$ , the increase in ELP is not that

significant [3]. Thus, it is always beneficial to keep the ELP threshold for multipath detection lower.

It can also be seen from these plots that both ELP and DELP provide similar increases in the presence of a given multipath. At some critical Doppler offsets, the peak for DELP is larger than that for ELP. However, this deviation is only relevant in absence of multipath, where a detection threshold has to be set. Thus, it can be concluded that although DELP is better than ELP in avoiding false alarm by lowering the detection threshold, their performance in the presence of multipath in the satellite signal being tracked is similar.



**Figure 9 – Comparison of maximum values of averaged ELP with that of DELP for PRN 12 in the presence of PRN 25 (IF=1405150 Hz) and multipath in PRN 12 signal with relative amplitude of 0.3 and phase difference of  $0.4918\pi$**



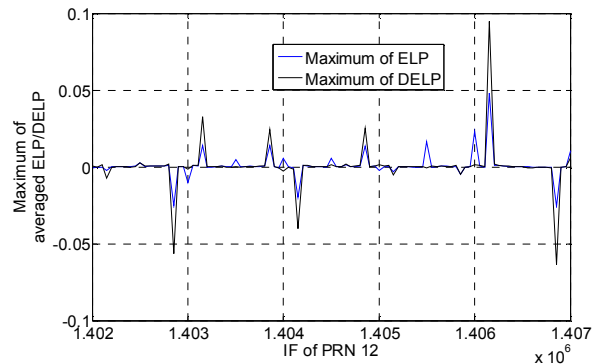
**Figure 10 – Comparison of minimum values of averaged ELP with that of DELP for PRN 12 in the presence of PRN 25 (IF=1405150 Hz) and multipath in PRN 12 signal with relative amplitude of 0.3 and phase difference of  $0.4918\pi$**

**Multipath in the interfering satellite signal:**

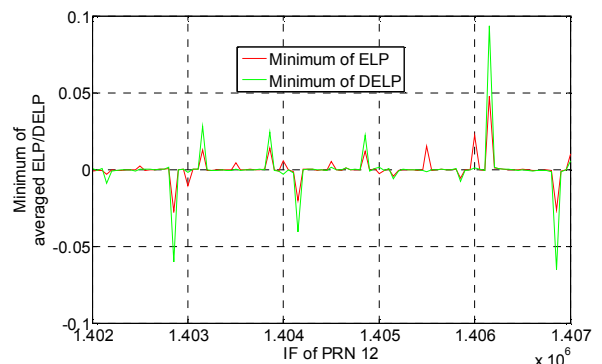
DELP at variable critical Doppler offsets is calculated using tracking loop outputs of the satellite being tracked and the interfering one. An error in these outputs can cause error in the ELP estimation and thus in DELP as well. In the presence of multipath in the signal of the interfering satellite, the carrier phase output of the

tracking loop for that signal has an error. This error is lower if the relative phase difference between the LOS signal and the reflected one is close to even integer multiples of  $\pi$ . Usually a receiver is not able to track a signal which has multipath with relative phase difference of odd integer multiples of  $\pi$ .

Figures 11 – 14 show the plots for the maximum and minimum of averaged ELP and DELP for PRN 12 in the presence of PRN 25 with multipath added to PRN 25 signal. Figure 11 and Figure 12 show the plots when multipath in PRN 25 signal has a relative phase difference of  $0.4918\pi$  with its LOS signal. It can be seen that at some frequencies, DELP is even worse than ELP as it has higher magnitude. However, ELP and DELP can detect multipath at any phase difference which is not integer multiple of  $\pi$ . Thus, in this case of where multipath in PRN 25 can be detected in its own ELP/DELP calculation, it may be concluded that ELP should be used instead of DELP at variable critical Doppler offsets whenever there is a multipath detected in the interfering satellite signal.



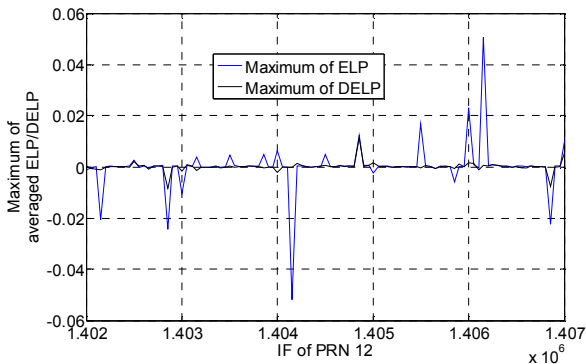
**Figure 11 – Comparison of maximum values of averaged ELP with that of DELP for PRN 12 in the presence of PRN 25 (IF=1405150 Hz) and multipath in PRN 25 signal with relative amplitude of 0.3 and phase difference of  $0.4918\pi$**



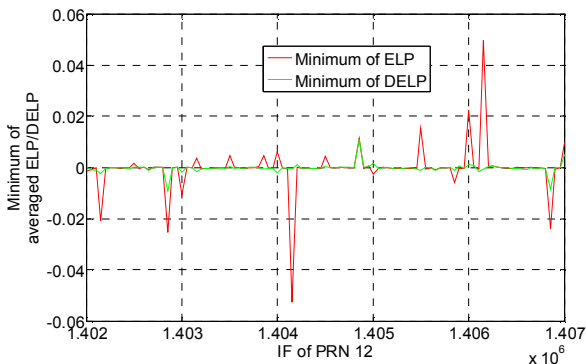
**Figure 12 – Comparison of minimum values of averaged ELP with that of DELP for PRN 12 in the presence of PRN 25 (IF=1405150 Hz) and multipath in PRN 25 signal with relative amplitude of 0.3 and phase difference of  $0.4918\pi$**

Now, let us consider a multipath in the interfering satellite signal with a relative phase difference close to an integer multiple of  $2\pi$  with its LOS signal. In this case ELP/DELP calculated in its own channel would not be able to detect multipath. Figure 13 and Figure 14 show the plots when multipath in PRN 25 signal has a relative phase difference of  $1.9672\pi$  with its LOS signal. It can be seen that DELP in this case is not affected by the multipath and its magnitude at variable critical Doppler offsets is still lower than the ELP.

Thus in this section, it has been shown that DELP is as effective as ELP in detecting multipath in the satellite signal being tracked. It has also been shown that in the presence of multipath in the interfering satellite signal, ELP is better than DELP whenever the relative phase difference of the reflected signal is not close to an integer multiple of  $\pi$ . However, in this case where multipath can be detected in the interfering satellite signal channel and ELP can be used instead of DELP at variable critical Doppler offsets in such scenario.



**Figure 13 – Comparison of maximum values of averaged ELP with that of DELP for PRN 12 in the presence of PRN 25 (IF=1405150 Hz) and multipath in PRN 25 signal with relative amplitude of 0.3 and phase difference of  $1.9672\pi$**



**Figure 14 – Comparison of minimum values of averaged ELP with that of DELP for PRN 12 in the presence of PRN 25 (IF=1405150 Hz) and multipath in PRN 25 signal with relative amplitude of 0.3 and phase difference of  $1.9672\pi$**

## CONCLUSION

This paper identifies fixed and variable critical Doppler offsets for multipath detection using ELP. It then proposes Differential ELP to be used at these Doppler offsets. DELP is a difference of ELP of the received signal and one estimated using the tracking loop outputs. ELP at fixed critical Doppler offsets has been estimated using a mathematical model presented earlier. A similar model for ELP at variable critical Doppler offsets has been proposed in this paper. It has been found that DELP is lower than ELP at these offsets and thus can have a lower threshold for multipath detection.

DELP has also been analyzed in the presence of multipath and it has been shown that DELP has an overall advantage over ELP. It has been found that DELP is as affected as ELP for detecting multipath. Moreover, whenever there is a multipath in the interfering satellite signal which can not be detected using ELP, i.e., the relative phase difference with the LOS signal is an integer multiple of  $2\pi$ , DELP is still better than ELP. However, when there is a multipath in the interfering satellite signal which can be detected using ELP, ELP should be used instead of DELP. It is because of the reason that DELP is calculated using tracking loop outputs, which has a phase error in the presence of the multipath.

## FUTURE WORK

It is intended to further improve the estimation of ELP in future. It can be done by improving the mathematical models for ELP estimation and modifying tracking loops to provide better estimate of the signal parameters. This will be helpful to further reduce the numerical value for DELP at critical Doppler offsets.

Moreover, the effect of multipath in the interfering satellite signal will be analyzed further to develop a robust algorithm to determine when ELP has to be used instead of DELP. It may also be possible to detect multipath in a given channel by analyzing the DELP in another channel.

## REFERENCES

1. Hatch, R., Knight, J., and Dai, L., "Recent GNSS Developments at NavCom Technology" in Proc. International Global Navigation Satellite Systems Society (IGNSS) Symposium, Sydney, December 4 – 6, 2007 paper 45
2. Mubarak, O. M., and Dempster, A., "Carrier phase analysis to mitigate multipath effect" in Proc. of the International Global Navigation Satellite Systems

Society (IGNSS) Symposium, Sydney, December 4 – 6, 2007 paper 64

3. Mubarak, O. M., “Analysis of Early Late Phase for Multipath Mitigation” (to appear) in Proc. of the 21<sup>st</sup> International Technical Meeting of the Satellite Division of the U.S. Institute of Navigation,

Savannah, Georgia, September 16 - 19, 2008  
(Available at: [http://www.gmat.unsw.edu.au/snap/publications/mubarak\\_2008a.pdf](http://www.gmat.unsw.edu.au/snap/publications/mubarak_2008a.pdf))

4. SoftGPS Project, 2006 <http://gps.aau.dk/softgps>