

Analysis of Early Late Phase for Multipath Mitigation

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BIOGRAPHY:

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ABSTRACT:

A lot of research has been done to reduce the adverse effects of multipath in order to provide accurate positioning in navigation systems; however it still remains one of the challenges. Most of research in the area of multipath mitigation has been focused on magnitude of correlator outputs ($I^2 + Q^2$), which does not take into account the phase delay between the line of sight (LOS) and reflected signals. Early late phase (ELP) proposed for the multipath detection and estimation exploits this phase difference. ELP is calculated as a phase difference between early and late correlator outputs. This phase difference is dependent on the attenuation of the reflected signal with respect to the LOS and time delay between the two. However, it has been shown in this paper that this phase difference is also dependant on various other controllable and uncontrollable parameters. The effect of these parameters needs to be analyzed in order to minimize false or missed detection of multipath. This provided motivation for the work presented in this paper. As ELP is computed using correlator outputs, these are first analyzed followed by ELP analysis. The effect of thermal noise, Doppler shift in carrier frequency, cross correlation, correlator spacing and ELP averaging period has been analyzed in this paper. The paper also presents mathematical models for theoretical evaluation of ELP in the absence and presence of multipath. Two kinds of mathematical models have been presented, one which is extensive in evaluation but provides exact estimation, and another which is easier to compute but provides a rougher

estimate. A comparison of both models and experimental results has also been presented.

1. INTRODUCTION:

Increasing use of GPS in urban and indoor applications during the last decade has been a motivation for research to reduce the positioning error induced by multipath in such environments. Different techniques have been presented to reduce this error, which include narrow correlators [1], multipath estimation technique (MET) [8], and Multipath estimating delay locked loop (MEDLL) [9]. In the presence of multipath, a reflected signal reaches the receiver after the line of sight (LOS) signal due to its longer path. It is because of this time delay that the two signals also have a carrier phase difference, unless the time delay is equal to an integer multiple of the carrier period [2]. However, all the above listed techniques and most other ones use the magnitude of the correlator outputs and hence do not exploit this phase difference. Recently, a novel variable, named early late phase (ELP) has been proposed for multipath detection and estimation by exploiting this phase difference [5]. In a standard receiver having three correlators (i.e., early, prompt and late), the prompt phase is always kept almost zero by the carrier tracking loop. The phase difference between the LOS and reflected signal would then be reflected in early and late correlator outputs. Therefore, ELP uses the difference between phases of the early and late correlators for multipath estimation. It is shown that ELP is quite effective in distinguishing the presence of multipath in a received signal in a noiseless environment. However, it needed to be analyzed in the presence of noise, cross correlation and different Doppler offsets in carrier frequency, so that it can be used in real environments. The effects on the ELP of these parameters, along with some others set at the receiver, are presented in this paper.

2. EARLY LATE PHASE:

In the absence of multipath, the energy at the correlator output is only present at one carrier phase, which is the phase of the carrier of incoming signal. Therefore, among the correlators in the I and Q channels, the I – channel gets

locked to the incoming carrier and thus at its output almost all the signal energy is present, whereas the Q – channel output has only noise. However, in the presence of one reflected signal in addition to the LOS, the correlator output has significant energy at two phases of the local carrier, one of the LOS signal and the other of the reflected one. Thus, the local carrier loop will lock to a phase somewhere in between these two phases. In this case, as the local carrier would neither be locked to the LOS nor reflected signal, there would be significant energy in the Q – channel output as well. This energy variation in I and Q channels due to multipath is shown in Figure 1. The experiments were run using simulated signals in the presence of thermal and cross correlation noise. The interfering satellites were given different Doppler shifts in carrier frequency, which is shown in brackets. The relative amplitude of the reflected signal with respect to the LOS one is given by α and is set to 1 for this experiment. The experiment is repeated for different values of Φ , which is the difference in carrier phase between LOS and reflected signals.

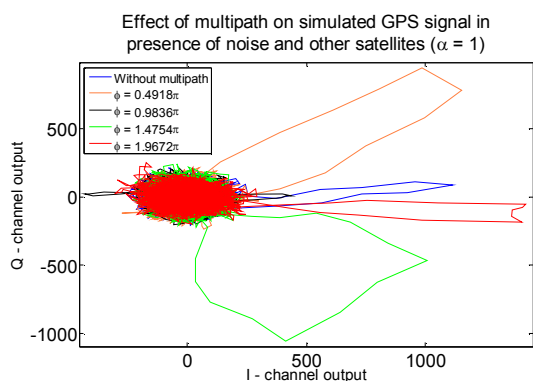


Figure 1 – PRN 7 in presence of PRN 12 (500 Hz), 15 (200 Hz), 19 (2000 Hz), 25 (3000 Hz), 28 (-2000 Hz), 31 (-3000 Hz) and 18 dB stronger noise – local carrier phase matched with LOS signal of PRN 7 [5]

In this experiment, it is assumed that the local carrier is locked to the LOS signal; however as mentioned above this is not possible in an actual receiver in the presence of multipath. In a receiver, the carrier tracking loop would keep maximum energy in the in-phase (I) channel of prompt correlator, or in other words would keep phase of prompt correlator output close to zero. That would mean that in the presence of multipath the prompt correlator phase would be around the center of the polygons shown in Figure 1. It further implies that the early and late correlators would then be sitting on shoulders of these polygons. Thus, in the presence of multipath the difference between the phase of early and late correlators would be relatively higher. Therefore, this phase difference has been named “early late phase” (ELP) and proposed for multipath detection and estimation. Mathematically, it is given by equation (1).

$$ELP(t) = \tan^{-1}\left(\frac{Q_L(t)}{I_L(t)}\right) - \tan^{-1}\left(\frac{Q_E(t)}{I_E(t)}\right) \quad (1)$$

$$\approx \tan^{-1}\left(\frac{Q_L(t)}{I_L(t)} - \frac{Q_E(t)}{I_E(t)}\right)$$

where I and Q correspond to I and Q channel, while subscripts E and L correspond to early and late correlator outputs, respectively. A time averaging filter can be applied on ELP to reduce the effect of thermal noise [5].

Numerical values of ELP in the presence and absence of multipath are shown in Figure 2 in the form of histograms for $\Phi \approx \pi/2$, which corresponds to an added path of around 4.76 cm for multipath at L1 carrier. It can be seen that ELP is quite effective in multipath detection and provides a good distinguishing feature for multipath presence even when the reflected signal amplitude is 30% of the LOS. Thus, a threshold can be set on the ELP value for detection of multipath and once detected, the current ELP value can be used for estimating relative strength and time delay of multipath. However, as shown in the following sections, ELP threshold for multipath detection is a function of various environmental conditions and receiver settings. Therefore, this paper analyzes the effect of these parameters on ELP so that this threshold may be estimated for a given set of parameters. This helps increase the robustness of this feature and avoid false or missed multipath detection alarms.

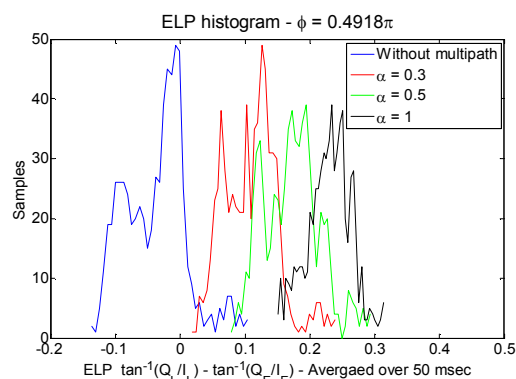


Figure 2 – Histogram for ELP output averaged over 50 msec of PRN 7 in presence of 18 dB noise and 7 other equally strong satellite signals (PRN: 3, 12, 15, 19, 23, 28 & 31) – $\Phi \approx 0.4918\pi$ [5]

3. ANALYSIS OF CORRELATOR OUTPUTS:

As ELP is computed from correlator outputs, before analyzing ELP these outputs first need to be analyzed. Figure 3 shows a basic block diagram of a GPS receiver. The RF front end brings down the carrier frequency from L1 to intermediate frequency (IF). In the presence of multipath, this frequency downconversion does not change the phase difference between the LOS and reflected signals, although the time delay is increased [4]. Therefore, phase delay is used in this paper instead of time delay when referring to relative delay of reflected signals

with respect to LOS, as later one gets changed after frequency downconversion.

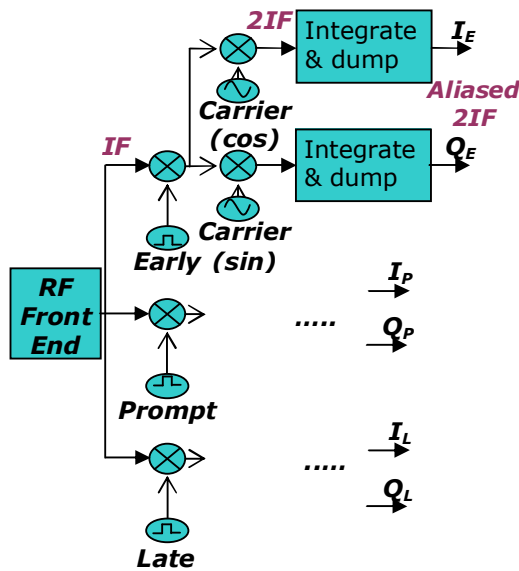


Figure 3 – Block diagram of a basic GPS receiver

After frequency downconversion, the signal is multiplied by the local carrier in the carrier tracking loop. Ideally the frequency of the local and incoming carriers would be equal to the IF. Thus, the result of this multiplication would have carrier frequency of 2IF. This is true for both I and Q – channels and also for early, prompt and late legs of the receiver.

Next is the integration and dump process. In order to get better insight of this process, it can be divided into two parts, integrating and downsampling. Integration is basically a low pass filter, so effectively it reduces the amplitude of the carrier frequency. Figure 4 shows the frequency response of a 1 msec integration filter. The IF of 1.405 MHz and sampling frequency of 5.714 MHz have been used which matches with those of Namuru, a GPS receiver platform developed at the University of New South Wales (UNSW), Australia [6]. The zoom in segments around DC and 2IF (2.81 MHz) show that the filter response at the DC is about 72 dB higher than that at 2IF. It further shows that at integer multiples of 1 kHz the response has nulls because integration over 1 msec of a carrier having frequency of 1 kHz or its integer multiple is zero.

Thus, the result of integration of a carrier is again a carrier of the same frequency but having reduced relative amplitude as compared to DC or lower frequencies present. The next step is to down-sample this carrier to 1 kHz. As the result of this, the 2IF frequency is aliased to less than or equal to 500 Hz. Moreover, when the IF is an integer multiple of 500 Hz, 2IF would be aliased to DC.

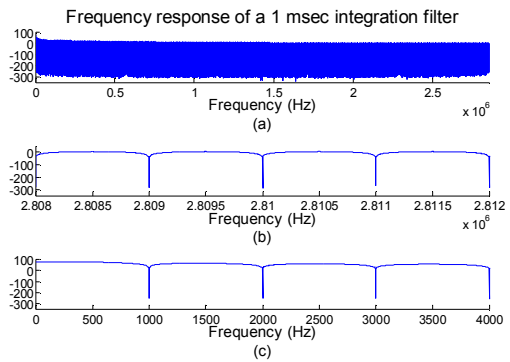


Figure 4 – (a) Frequency response of a 1 msec integration filter (b) Zoom in around 2.81 MHz (c) Zoom in around DC

In a receiver, the integrate and dump function is performed in one go rather than these two steps, although the result of these two are same but far less processing is required if it is performed as a single process. The outputs of correlators from a software receiver [7] are shown in Figure 5, which confirms the theory presented above. A simulated noiseless signal is used for the experiment with IF set to $1.405 \times 10^6 + 50$ Hz. This means that the 2IF would be $2.81 \times 10^6 + 100$ Hz and after integrate and dump process it would be aliased to 100 Hz. Thus, in all correlator outputs 100 Hz carrier is present, although its mean and amplitude varies depending on local code phase (different for early, prompt and late) and local carrier phase (different for I and Q – channel) for particular output. Although this attenuated residual carrier generally does not have much effect in tracking loops, it is shown in next section that it has to be considered in order to analyze ELP in various conditions.

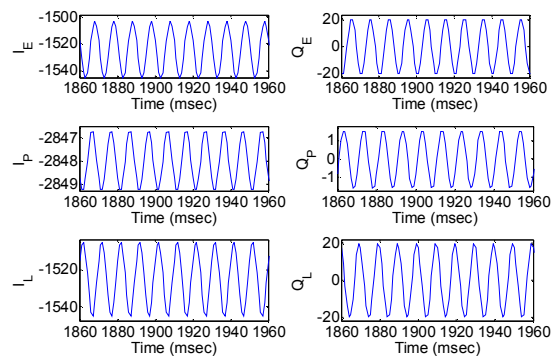


Figure 5 – Correlator outputs from a software receiver for IF = $1.405 \times 10^6 + 50$ Hz

4. ANALYSIS OF ELP:

In this section, ELP is analyzed in the absence and presence of multipath. This analysis is helpful in determining a threshold on ELP value, beyond which a multipath presence alarm could be raised. As mentioned

above, this threshold could be a function of different parameters, which is analyzed in this section.

As can be seen from equation (1), ELP is calculated using correlator outputs and as these outputs have residual carrier of aliased 2IF frequency in them, ELP also has the same frequency component. Figure 6 shows ELP in a noiseless environment and absence of multipath for different Doppler shifts in carrier frequency, or in other words different values of IF. It can be seen that the ELP is a sinusoid of 2IF frequency aliased to less than 500 Hz with some sampling noise. For example, for the IF of $1.405 \times 10^6 + 10$ Hz, the ELP is a 20 Hz wave (period of 50 msec) and for IF of $1.405 \times 10^6 + 500$ Hz the ELP is a DC value as IF is an integer multiple of 500 Hz.

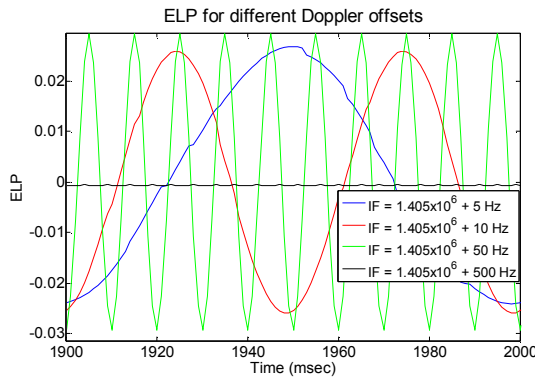


Figure 6 – ELP for different intermediate frequencies in absence of multipath

Now let us see the effect of multipath on ELP. Figure 7 shows ELP in the same conditions as above but in the presence of multipath. It can be seen that the ELP is still a sinusoid of the same frequency and amplitude, however, there has been a DC offset because of multipath. This DC offset is dependent on the relative amplitude and time delay of the reflected signal as compared to the LOS signal, and thus can be used for multipath detection and estimation.

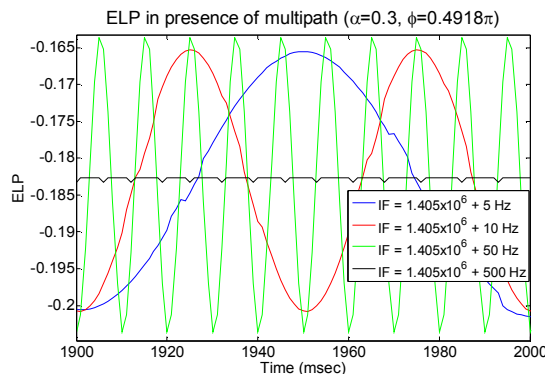


Figure 7 – ELP for different intermediate frequencies in presence of multipath

Apart from multipath, there are parameters which effect ELP value and they can be divided into two categories,

namely controllable and uncontrollable parameters. Controllable parameters are those which can be set and changed at the receiver, while uncontrollable parameters are those which are not under the receiver's designer or user control and although they may be known at a given time they cannot be set to a desired value. The parameters from both categories are listed in Figure 8 and their effect on ELP will now be addressed individually.

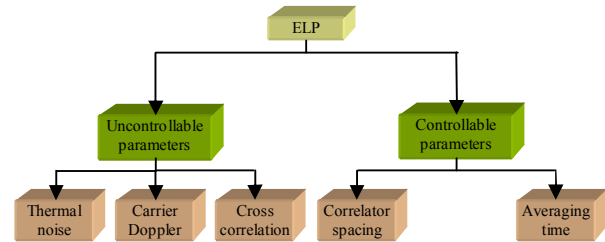


Figure 8 – Parameters effecting ELP in absence of multipath

4.1 Effect of thermal noise

In the presence of thermal noise, the ELP deviates around the value obtained in a noiseless environment. This deviation is higher for stronger noise. In any case, this can be significantly reduced by averaging ELP over time. It has been shown that averaging ELP over 50 msec can effectively reduce the noise effect [5]. However, averaging time may be reduced if the receiver is moving with respect to the surroundings and for environments having higher signal to noise ratio.

4.2 Effect of Doppler offset in carrier frequency

As shown in Figure 6, ELP gives a sinusoid over time whose frequency is dependent on the Doppler offset in carrier frequency. Moreover, in order to reduce the effect of thermal noise on ELP, time averaging is performed as mentioned above. In the absence of multipath, the result of this averaging is almost zero because a sinusoid having a mean value of zero is being averaged. However, when the IF is an integer multiple of 500 Hz, ELP is a non-zero DC value instead of a sinusoid, so its average is also a nonzero DC value. In other words, time averaging is a low pass filter and removes all the frequencies present in ELP except DC or very low frequencies. Thus, at the Doppler offsets in the carrier frequency which give such an IF, the ELP value would be higher than is obtained otherwise. Figure 9 shows the plot for ELP obtained from a software receiver using a noiseless simulated signal of PRN 7. ELP is calculated every 1 msec and averaged over 20 msec. The minimum and maximum of this averaged ELP are computed from a few seconds interval and plotted for different Doppler offsets in carrier frequency. It can be seen that there are peaks at each IF which is integer multiple of 500 Hz. The height of these peaks is determined by spectral code lines, which are modulated by IF. Therefore, at each 500 Hz integer multiples the maximum value of peaks is different and these values are different for each PRN code. Figure 10 shows the same

plot for PRN 12. It can be seen that although it also has peaks at 500 Hz integer multiple frequencies, their height at a given IF is quite different from those obtained for PRN 7 in Figure 9.

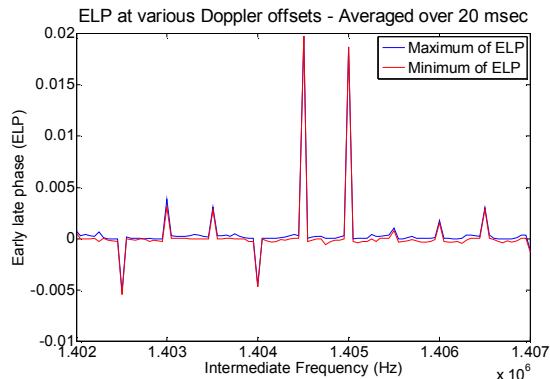


Figure 9 – ELP averaged over 20 msec in absence of multipath (PRN 7)

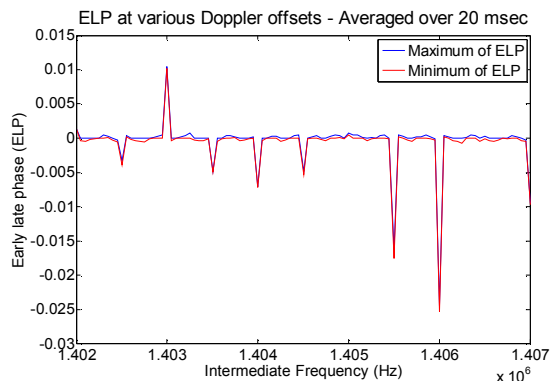


Figure 10 – ELP averaged over 20 msec in absence of multipath (PRN 12)

As shown in Figure 7, the frequency of the residual carrier in ELP remains the same in the presence of multipath. Thus, although generally the ELP value is higher in the presence of multipath, at 500 Hz integer multiple frequencies similar peaks are obtained. Figure 11 shows a plot for averaged ELP in the presence of multipath, where relative amplitude of the reflected signal as compared to LOS is ($\alpha=$) 0.3 and the phase difference between the two is ($\Phi=$) 0.4918π . The pattern of these peaks is quite similar to one obtained without multipath in Figure 9, although there are some differences as well. However, the main difference is the rise in overall values by around 0.17. The amount of this rise is higher for higher values of α . It is also dependant on Φ , as it is higher for Φ closer to $\pi/2$ and lower for Φ close to 0 or π . Moreover, ELP is moves towards being negative instead when Φ is between π and 0. The presence of the peaks in Figure 11 highlights the difficulty in setting an ELP threshold.

Thus, from the above analysis it can be said that a positive and a negative limit can be set on ELP value to detect multipath. These limits should be close to zero to avoid missed multipath detection, however a higher threshold

limit should be set where IF is close or equal to 500 Hz integer multiple to avoid false alarm of multipath detection.

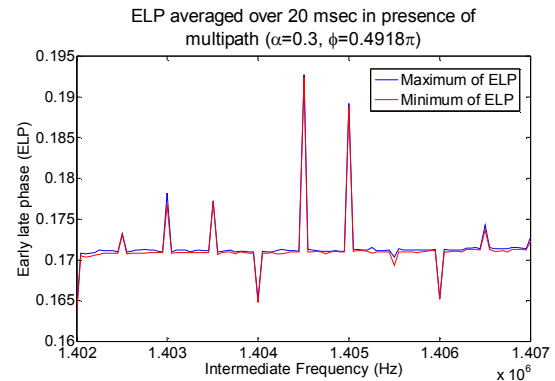


Figure 11 – ELP averaged over 20 msec in presence of multipath (PRN 7)

4.3 Effect of cross correlation

Figures 9 – 11 analyzed ELP in a noiseless environment. However, the real time output of an RF front end in a GPS receiver consists of around 6 – 8 satellite signals, which means that for each channel there are at least 5 satellite signals producing cross correlation. This provides motivation to analyze the effect of cross correlation on ELP.

The effect of an interfering satellite is analyzed, which is then generalized to any number of satellites present. If the IF of interfering satellite is given by f_1 and that of tracking satellite by f_2 and the phase difference between the two by ψ , then the input of integrate and dump can be given by equation (2).

$$d(t) = \cos(2\pi f_1 t) * \cos(2\pi f_2 t + \psi) * x(t) \\ = 1/2 [\cos(2\pi(f_1 - f_2)t - \psi) - \cos(2\pi(f_1 + f_2)t + \psi)] * x(t) \quad (2)$$

where $x(t)$ is the cross correlation of codes of two satellites at a given phase difference. It can be seen from this equation that the output of correlators would have difference and sum of two frequencies aliased to 500 Hz due to the integrate and dump process, as explained above. These frequencies would also then be present in ELP in addition to the residual carrier of the satellite being tracked. Figure 12 shows the frequency spectrum of ELP for PRN 12 in the presence of an equally strong PRN 25 signal in an otherwise noiseless environment. The IF of PRN 12 was set at 1405025 Hz, which gets doubled to 2810050 Hz after passing through the correlator and then aliased to 50 Hz after passing through the integrate and dump working at 1 kHz (integration period of 1 msec). Thus, a 50 Hz component in the figure is because of the residual carrier of PRN 12. The IF of PRN 25 was set at 1405150 Hz. As mentioned above, ELP would also have two other frequencies, one equal to the difference in IF of

these signals and other sums of them aliased to 500 Hz. The difference of the two is 125 Hz, which can be seen in Figure 12. The sum is 2810175 Hz, which is aliased to 175 Hz and can be seen in the ELP spectrum as well.

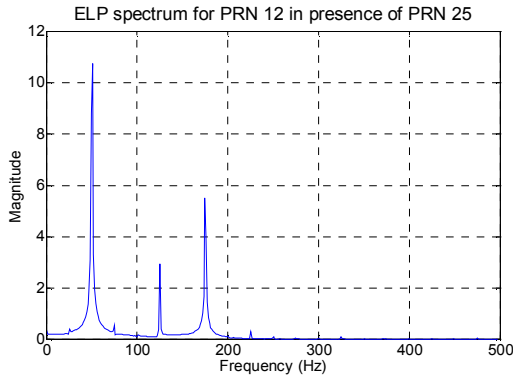


Figure 12 – ELP spectrum for PRN 12 (IF=1405025 Hz) in presence of PRN 25 (IF=1405150 Hz)

While analyzing the effect of Doppler shift, it was seen that when residual carrier frequency is close to zero, the value for averaged ELP gets significantly higher. Similarly, in this case when either the aliased sum or difference of interfering and tracking satellites carrier frequencies is close to zero, the magnitude of averaged ELP gets higher. The sum of two frequencies would be aliased to DC whenever it is an integer multiple of 1 kHz.

The effect of Doppler offset on PRN 12 was shown in Figure 10. Now, a PRN 25 signal with IF of 1405150 Hz has also been added to that signal in order to experimentally confirm the effect of cross correlation. The resultant plot is given in Figure 13. It can be seen that in addition to the peaks present in Figure 10, there are peaks present at Doppler offsets where either difference or sum between the IF of PRN 12 and PRN 25 is equal to an integer multiple of 1000 Hz. The heights of the peaks in Figure 10 were determined by PRN code and Doppler shift in carrier frequency. Similarly, the height of these additional peaks is determined by the PRN codes of the two signals and their relative Doppler offsets.

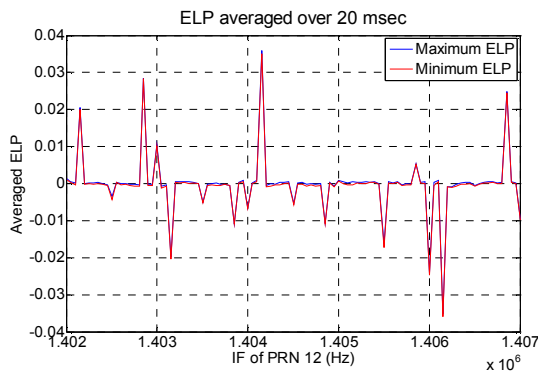


Figure 13 – ELP for PRN 12 averaged over 20 msec in presence of PRN 25 (IF=1405150 Hz) and absence of multipath

Earlier it has been established that the ELP threshold for multipath detection should be higher at each IF which is an integer multiple of 500 Hz. From the above cross correlation analysis, it may also be added that this threshold should also be higher at each IF which if added or subtracted from the IF of any other satellite signal present gives an integer multiple of 1 kHz. The IF for other satellites present can be found from other channels of the receiver.

4.4 Effect of correlator spacing

As ELP is the phase difference between outputs of early and late correlators, the effect of the spacing of these two correlators from the prompt is also worth investigating. Usually this spacing is set to 0.5 chips, however a smaller spacing is used for reduced tracking error in the presence of multipath [1]. In the absence of multipath, change in correlator spacing does not have much effect because the phase difference between the early and late correlator outputs is close to zero anyway. However, the correlator spacing has an effect at the peaks observed during the discussion of Doppler offsets and cross correlation. Figure 14 shows ELP averaged over 50 msec zoomed around IF of 1.404 MHz for PRN 7. The difference between maximum and minimum ELP gives the amplitude of the residual carrier sinusoid. This difference goes to zero at 1.404 MHz as there is only DC at this point instead of any residual carrier. It can be seen from the plot that the ELP value is not changed much when correlator spacing is reduced from 0.5 chips to 0.2 chips. Thus, it can be said that the threshold for multipath detection is not dependant on correlator spacing.

In the presence of multipath, early and late correlators sit on the shoulders of the polygons shown in Figure 1. Higher correlator spacing implies larger distance between the two and thus a higher value for ELP for a given multipath. As the shift in ELP due to the presence of multipath is used to detect it, therefore it is even tougher to detect multipath for smaller correlator spacing.

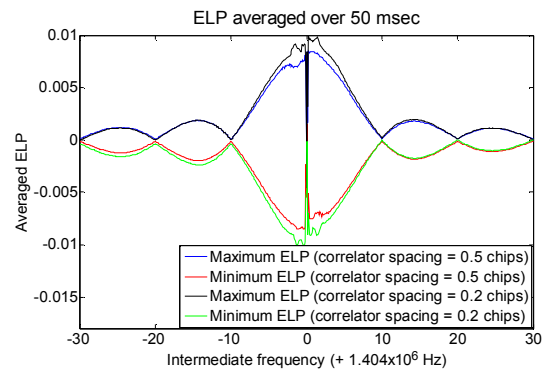


Figure 14 – ELP in the absence of multipath for PRN 7 averaged at 50 msec and zoomed to ± 30 Hz around 1.404 MHz

Figure 15 shows averaged ELP in the presence of multipath for 0.2 chips correlator spacing. The multipath

and PRN are the same as in Figure 11. It can be seen that due to reduction in correlator spacing from 0.5 chips in Figure 11 to 0.2 chips here, the approximate mean value of ELP has been shifted down from 0.17 to 0.113. Thus, while narrow correlators are generally useful for reducing tracking error caused by the multipath, they should not be used for ELP calculation as they reduce the variation in its value due to presence of multipath and thus reduce the probability of multipath detection.

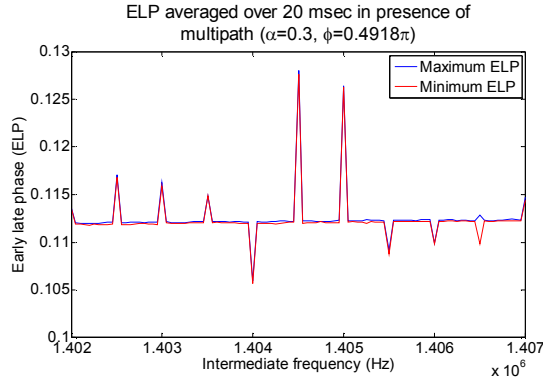


Figure 15 – ELP averaged over 20 msec in presence of multipath for 0.2 chips of correlator spacing (PRN 7)

4.5 Effect of ELP averaging time

As mentioned in section 4.1, ELP is averaged in time to reduce the effect of thermal noise. Averaging duration is determined by SNR and receiver movement. Longer averaging duration should be used in lower SNR environments. Moreover, smaller averaging duration is preferred where the receiver is moving at high speed. Thus, it could be a compromise between these two factors and should be adjusted accordingly.

In a noiseless and stationary environment, the ELP averaging time only determines the frequency of sinc functions formed around each IF of 500 Hz multiple as shown in Figure 14. Figure 16 compares the sinc formed with 0.5 chips spacing for 20 msec and 50 msec integration period. It can be seen that a wider sinc wave is obtained for lower averaging duration. Thus, the threshold for multipath detection has to be adjusted accordingly around the peaks formed at 500 Hz integer multiples IF.

5. MATHEMATICAL EVALUATION OF ELP:

In this section, ELP is evaluated mathematically in a noiseless environment and then compared with experimental results. ELP is evaluated in the absence and then presence of multipath.

5.1 Without multipath

As discussed in section 3, there is a residual carrier in each of correlator outputs. First, let us consider the prompt correlator output assuming that local and received PRN codes and carriers are perfectly aligned. As a result the

summation for the Q-channel can be given by equation (3).

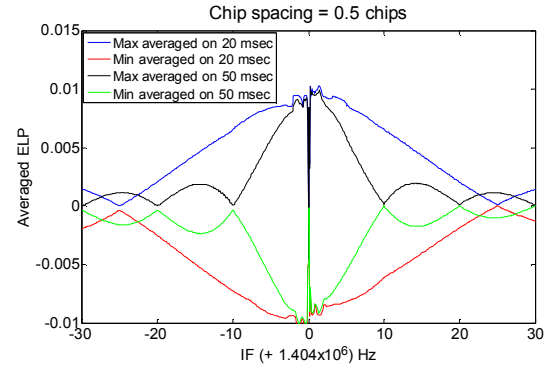


Figure 16 – ELP in the absence of multipath for PRN 7 zoomed to ± 30 Hz around 1.404 MHz

$$\begin{aligned}
 Q_P(x) &= \frac{1}{2} \sum_{k=x}^{x+(s-1)} \sin\left(\frac{2\pi(2f)}{FS} k\right) \\
 &= \frac{1}{2} \left[\sin\left(\frac{4\pi f}{FS} x\right) \sum_{k=0}^{s-1} \cos\left(\frac{4\pi f}{FS} k\right) + \cos\left(\frac{4\pi f}{FS} x\right) \sum_{k=0}^{s-1} \sin\left(\frac{4\pi f}{FS} k\right) \right] \\
 &= \frac{1}{2} \left[\sin\left(\frac{4\pi f}{FS} x\right) \frac{\sin\left(\frac{2\pi f}{FS} s\right) \cos\left(\frac{2\pi f}{FS} (s-1)\right)}{\sin\left(\frac{2\pi f}{FS}\right)} \right. \\
 &\quad \left. + \cos\left(\frac{4\pi f}{FS} x\right) \frac{\sin\left(\frac{2\pi f}{FS} s\right) \sin\left(\frac{2\pi f}{FS} (s-1)\right)}{\sin\left(\frac{2\pi f}{FS}\right)} \right] \quad (3)
 \end{aligned}$$

where f is the IF, FS is the sampling frequency and s is the number of samples per integration interval. It can be seen that as the result of summation, the frequency of the signal remains same; however the signal is scaled and delayed in phase. The result of the integrate and dump is $Q_P(x)$ sub-sampled at 1 kHz, which results in aliasing of f to within a 500 Hz range. This provides a mathematical explanation of the residual carrier presence in correlator outputs as explained in section 3.

A similar equation for early and late correlator outputs is a bit more complicated as PRN code is not stripped out completely. The output of the late correlator in the Q channel for a PRN code $c(k)$ can be given by equation (4).

$$Q_L(x) = \sum_{k=0}^{s-1} \left[\cos\left(\frac{2\pi f}{FS} (x+k)\right) \sin\left(\frac{2\pi f}{FS} (x+k)+\theta\right) c(x+k) c(x+k-\varepsilon) \right]$$

$$= \frac{1}{2} \sum_{k=0}^{s-1} \left[\left(\sin(\theta) + \sin\left(\frac{4\pi f}{FS}(x+k) + \theta\right) \right) c(x+k)c(x+k-\varepsilon) \right] \quad (4)$$

where ε is the correlator spacing and θ is the carrier phase error, which should ideally be equal to zero. Although summation can be computed offline, a simpler solution can be obtained if integration is used instead of summation. An indefinite integral only gives an approximate constant ELP and does not provide residual carrier at the output. However, as a simpler equation would be easier to implement in receiver, an integration solution is also provided here.

In terms of the integral, the phase of late correlator can be given by equation (5).

$$\tan^{-1}\left(\frac{Q_L(t)}{I_L(t)}\right) = \tan^{-1}\left(\frac{\int \cos(2\pi ft) \sin(2\pi ft + \theta) c(t)c(t-\varepsilon) dt}{\int \cos(2\pi ft) \cos(2\pi ft + \theta) c(t)c(t-\varepsilon) dt}\right)$$

$$= \tan^{-1}\left(\frac{\sin(4\pi ft + \theta)R(\varepsilon) - 4\pi f \int \cos(4\pi ft + \theta)R(\varepsilon) dt}{\cos(4\pi ft + \theta)R(\varepsilon) + 4\pi f \int \sin(4\pi ft + \theta)R(\varepsilon) dt}\right)$$

$$= \tan^{-1}\left(\frac{+\sin(\theta)R(\varepsilon)}{+\cos(\theta)R(\varepsilon)}\right) \quad (5)$$

where $R(\varepsilon)$ is the autocorrelation function of the PRN code. As the autocorrelation function is symmetrical around $\varepsilon=0$, ELP would be zero in the absence of multipath.

$$R(\varepsilon) = R(-\varepsilon) \Rightarrow \frac{Q_E(t)}{I_E(t)} = \frac{Q_L(t)}{I_L(t)} \Rightarrow ELP(t) = 0 \quad (6)$$

Figure 17 shows a comparison between experimental and estimated ELP values. The estimated sinusoid is obtained using summation, as given in equation (4). It can be seen that this estimated sinusoid is almost equal to the actual one. The plot also shows a zero value obtained from equation (6), which does not have the residual carrier; however it does provide a mean of actual ELP.

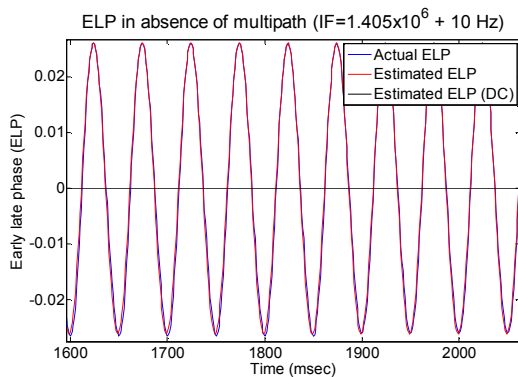


Figure 17 – Comparison of estimated and actual values of ELP in absence of multipath

5.2 With multipath

In the presence of multipath, a delayed and attenuated form of signal is also present in the received signal. Moreover, the phase difference between the LOS and prompt correlator would not be zero; instead it will be equal to the tracking error due to the multipath. As a result, the difference between the code phase of the LOS and the early code would be reduced and that with the late code would be increased, although the difference between early, prompt and late would remain same.

For a time delay of τ between LOS and reflected signals and relative attenuation of α , equation (4) would be modified to equation (7) below in presence of multipath.

$$Q_L(x) = \sum_{k=0}^{s-1} \left[\begin{array}{l} \cos\left(\frac{2\pi f}{FS}(x+k)\right)c(x+k) + \\ \alpha \cos\left(\frac{2\pi f}{FS}(x+k-\tau)\right)c(x+k-\tau) \\ \sin\left(\frac{2\pi f}{FS}(x+k) + \theta\right)c(x+k-\varepsilon-d) \end{array} \right] \quad (7)$$

where d is the tracking error because of the multipath and can be found for given multipath conditions and receiver parameters [4]. Again, although equation (7) can be implemented in software, a simpler equation can be obtained using indefinite integration to obtain an approximate DC value without a residual carrier. Thus, equation (5) can be modified to include multipath as follows.

$$\tan^{-1}\left(\frac{Q_L(t)}{I_L(t)}\right) = \tan^{-1}\left(\frac{\int \left(\cos(2\pi ft)c(t) + \alpha \cos(2\pi f(t-\tau))c(t-\tau) \right) \sin(2\pi ft + \theta)c(t-\varepsilon-d) dt}{\int \left(\cos(2\pi ft)c(t) + \alpha \cos(2\pi f(t-\tau))c(t-\tau) \right) \cos(2\pi ft + \theta)c(t-\varepsilon-d) dt}\right) \quad (8)$$

This equation can be simplified using integration by parts and can be given by equation (9).

$$\tan^{-1}\left(\frac{Q_L(t)}{I_L(t)}\right) = \tan^{-1}\left(\frac{\sin(\theta)R(\varepsilon+d) + \alpha \sin(\theta + 2\pi f\tau)R(\varepsilon+d-\tau)}{\cos(\theta)R(\varepsilon+d) + \alpha \cos(\theta + 2\pi f\tau)R(\varepsilon+d-\tau)}\right) \quad (9)$$

Similarly, the phase of early correlator output in presence of multipath can be given by equation (10).

$$\tan^{-1}\left(\frac{Q_E(t)}{I_E(t)}\right) = \tan^{-1}\left(\frac{\sin(\theta)R(-\varepsilon+d) + \alpha \sin(\theta + 2\pi f\tau)R(-\varepsilon+d-\tau)}{\cos(\theta)R(-\varepsilon+d) + \alpha \cos(\theta + 2\pi f\tau)R(-\varepsilon+d-\tau)}\right) \quad (10)$$

It can be seen that ELP, which is the difference of these two phases would not be zero here. Instead an approximate DC ELP value can be computed from these equations.

The autocorrelation function R used in equation (9) and (10) can be given by equation (11) [3].

$$R(\delta) = A^2 \left(1 - \frac{|\delta|}{T_c} \right) \quad \text{for } |\delta| \leq T_c \quad (11)$$

$$= 0 \quad \text{elsewhere}$$

where A is the amplitude of PRN code, which is equal to 1 for GPS codes and T_c is the chipping rate for the code. In the presence of multipath, carrier phase error θ is given by equation (12) [4].

$$\theta = \arctan \left(\frac{\alpha R(\tau) \sin(\varphi)}{1 + \alpha R(\tau) \cos(\varphi)} \right) \quad (12)$$

Figure 18 shows a comparison between experimental and estimated values of ELP in the presence of multipath. Again the estimated sinusoid is computed using summation equations for each correlator output, like equation (7). An approximated DC value is found using difference of equation (9) and (10).

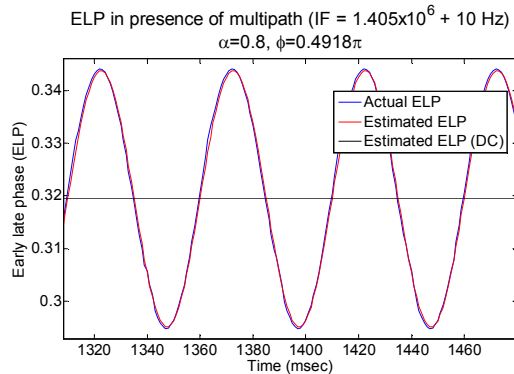


Figure 18 – Comparison of estimated and actual values of ELP in presence of multipath

It has been shown in this section that ELP can be accurately estimated for given set of conditions in the presence and absence of multipath, using a summation equation. It has also been found that an approximate ELP value can also be found using a simpler equation, which is easy to compute in a receiver.

CONCLUSION:

This paper presented an analysis of early late phase (ELP) for effective multipath detection and estimation. It has first presented an analysis of correlator outputs in noiseless environment and showed that a residual carrier is present in all correlator outputs of a GPS receiver. This residual carrier is then found in the ELP as well. It also

showed that in the presence of another satellite signal, two additional frequency components are also present in the signal which are the aliased sum and difference of frequencies of satellite carriers. As a result of this analysis, it has been concluded that a higher ELP threshold is required for multipath detection at each IF which is an integer multiple of 500 Hz or whose difference or sum with IF of any other satellite present is equal to integer multiple of 1 kHz.

The effect of ELP averaging time and correlator spacing on ELP were also analysed. It was shown that ELP averaging time determines the width of sinc functions formed around each IF of 500 Hz multiple. Moreover, in the absence of multipath, there is not much effect of correlator spacing, however it is tougher to detect a given multipath for smaller correlator spacing.

The paper also presented two mathematical models for evaluation of ELP. One is extensive in evaluation and provides exact estimation and the other is simple to compute but provides an approximate mean value. A comparison of both models was presented with experimental results in the presence and absence of multipath.

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