

Generating a 3D TEC Model for Australia with Combined LEO Satellite and ground base GPS Data

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BIOGRAPHY

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Dr Jinling Wang is a Senior Lecturer in the School of Surveying and Spatial Information Systems, UNSW. He is a Fellow of the Royal Institute of Navigation, UK, and a Fellow of the International Association (IAG) of Geodesy. Jinling is a member of the Editorial Board for the international journal GPS Solutions, and Chairman of the study group (2003-2007) on pseudolite applications in positioning and navigation within the IAG's Commission 4. He was 2004 President of the International Association of Chinese Professionals in Global Positioning Systems (CPGPS). Jinling holds a PhD in GPS/Geodesy from the Curtin University of Technology, Australia.

Dr David Cole is former director of IPS (Ionospheric Prediction Service) Radio and Space Services of Department of Industry Tourism and Resources of Australia. He has led IPS over the past two decades. He is an expert in the field of radio propagation and space weather. He is Chairman of the International Study Group on radio wave propagation within the International Telecommunication Union.

ABSTRACT

Characteristics of temporal variations of ionospheric TEC (Total Electron Content) play an important role, in GPS

(Global Positioning System) static, kinematic and navigation application, especially in the derivation of high precision ionospheric corrections for position. There are many models advanced by researchers for modeling these characteristics. But most of them are 2 dimensional space ionospheric models, they only consider latitude and longitude as variables. In this paper, we present a near real-time 3 dimensional TEC calculation method in which latitude, longitude, altitude and time are treated as the parameters. In the following sections, four topics are discussed: Firstly, generating vertical electron density distribution function by LEO (Low Earth Orbit) RO (Radio Occultation) data; Secondly, calculating "less bias" VTEC (vertical TEC) value by carrier phase smoothing pseudo ranges using Kalman filter method and "no bias" TEC linear model assumption method; Thirdly, the VTEC response to different heights formed by using TEC vertical distribution profile as a proportion rate, and Fourthly, some initial experiment results.

INTRODUCTION

The ionosphere, mainly generated by extreme ultraviolet (EUV) light and X-ray from the Sun, locates at about from 50 kilometers to 2500 kilometers above the earth. It consists of D, E, F1 and F2 layers. The F2 layer has maximum electron density and contributes to the biggest GPS signals effect. The upper boundary (above 1000 kilometers) of the ionosphere is not well defined because the electron distribution is thin in this area. This layer also is described as plasmasphere or protonosphere in which positive ions (protons) are dominant (Langley, 2000). The ionosphere is the largest component deducing the accuracy of GPS positioning. Therefore, it is necessary to study the ionospheric profile in 3 dimensional space, taking into account real time distributions and features. The data from ground based GPS receivers and LEO GPS receivers can be effectively used to achieve these targets (Kaplan, 1996).

In early time, only ground based GPS system was adopted to analyze ionosphere. According to these ground based GPS models, the ionosphere is treated as an infinite thin layer located at about 250 kilometers above the surface of

the Earth, and the slant TEC is defined as the total electron content, along the ray paths from ground based receivers to GPS satellites. The slant TEC is calculated as:

$$TEC_{slant} = \frac{f_1 f_2}{40.3(f_2^2 - f_1^2)} (L_1 - L_2) + Bias,$$

in which f_1 and f_2 are GPS satellite frequencies ($f_1 = 1575.42\text{MH}$, $f_2 = 1227.6\text{MH}$). L_1 and L_2 are measured carrier phase by dual-frequency GPS receivers. The *Bias* is the sum of hardware error, multipath error and measurement noise error (Liu, et al., 2005). For getting unambiguous L_1 , L_2 and solving the *Bias*, many different methods have been researched (Gao et al., 2002). But based on these models, TEC is as the total amount along the ray path from ground receivers to GPS satellites, it is impossible to divide TEC value to different layers response to different altitude, and further more to build up a near real time 3 dimensional TEC distribution models regarding latitude, longitude, altitude and time etc. as parameters.

The RO technique has been developed since 1960s by JPL (Jet Propulsion Laboratory) and Standford University. Initially, the purpose was to research other planets. From 1995, RO technique has combined GPS to study the Earth's atmosphere and the ionosphere. GPS receivers are on board to LEO satellites (i.e. COSMIC, CHAMP, SAC-C and GRACE) and forward and backward real time sounding GPS signals or RO (Hajj, et al., 2002). The GPS RO system consists of GPS receivers, high-gain occultation antennas and precise orbit determination (POD) antennas. About 2,500 globally distributed RO are sounded every day. When GPS radio signals pass through the ionosphere and atmosphere, they are refracted and/or diffracted. The result is that the bending angles and the phase variation are generated. Through calculating these bending angles and excess Doppler frequency shift, the vertical electron density profile response to different layers are generated, and further more the TEC vertical distribution profile is acquired.

METHODOLOGY

Figure 1 gives a flow chat on the process to generate 3D ionospheric model. Firstly, LEO RO data are processed, then electron densities of different layers are formed and further more, a vertical electron density function response to height is generated. Secondly, ground based GPS data are processed and 2D ionospheric model response to time, latitude and longitude variables is generated. Thirdly, TEC vertical distribution profile is produced by vertical electron density function. Through combination TEC vertical distribution profile and ground based 2D model, A 3D ionospheric model with time, latitude, longitude and altitude as variables is acquired.

Generating Vertical Electron Density Function

When GPS radio signals pass through the ionosphere, the bending angles are very small (about 0.03°), especially in F2 layer and more upper layers. Although these small bending angles indeed cause a certain mis-modelling in E layer, which might be ignored, because it is much smaller amount than the bias caused by the assumption that the electron density in a large local area around the ray path tangent point (the perigee to the Earth) is local spherical symmetry.

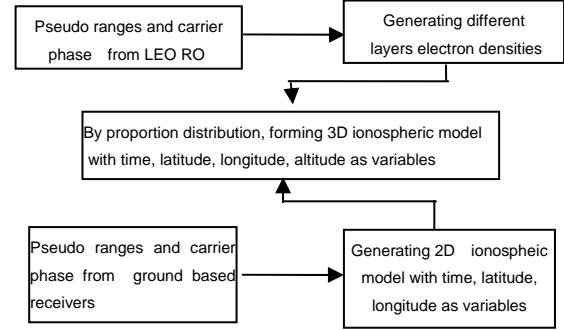


Figure 1: Flow chart of forming 3D ionospheric model

Therefore, it is assumed that GPS radio signals propagate along a straight line for GPS satellites to GPS receivers boarded on LEO satellites.

In this paper, LEO COSMIC satellites data were used to generate 3 dimensional Australia region TEC model result. Because the COSMIC satellites initial orbit is about 500km, the ionosphere is divided (from 50 kilometers to 1000 kilometers and extending to more) to 10 layers. They are 50km~100km, 100km~150km, 150km~200km, 200km~250km, 250km~300km, 300km~350km, 350km~400km, 400km~450km, 450km~500km and 500km~unlimited boundary with corresponding electron densities N_i ($i = 10$ to 1). We assume that the electrons evenly distribute in each layer. Also, we assume electron density is zero in 0~50km (this layer is defined as troposphere). We use recursive Abel inversion algorithm with the TEC discretization to get the electron density in each layer. If the radio signal goes through i^{th} layer, it must pass from 1 to $(i-1)^{th}$ layers (refer to **Figure 2**). We denote TEC_i as the TEC values when the lowest layer near the Earth, in which the signal goes through, is i^{th} layer. Then

$$TEC_i = 2 \cdot \sum_{j=1}^i N_j \cdot l_{j,i} \quad (1)$$

Or

$$N_i \equiv \frac{TEC_i - 2 \cdot \sum_{j=1}^{i-1} N_j \cdot l_{j,i}}{2 \cdot l_{i,i}} \quad (2)$$

where TEC_i is the total electron content along the ray path from GPS to LEO satellites, and l_{ji} is the length of the ray path between two ionospheric layers

Then we are able to calculate N_i ($i = 1$ to 10) starting from the utmost layer (or layer 1) to the bottom layer (layer 10) by “peeling onion” method. (Syndergaard et al., GarciaFernandez, 2004; Hernandez-Pajares et al., 2000).

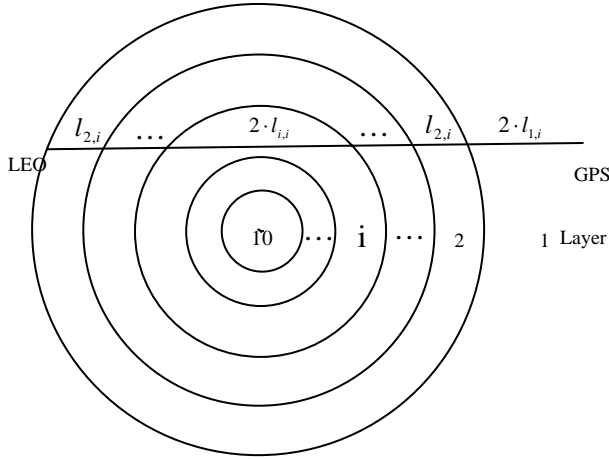


Figure 2: RO and 10 layers Ionosphere

For example, based on Equation (2), layer 1 electron density is expressed as below.

$$N_1 = \frac{TEC_1}{2 \cdot l_{1,1}} \quad (3)$$

where N_1 is the electron density of layer 1 (from 500km~unlimited boundary), $2 \cdot l_{1,1}$ is the distance from the LEO satellite to the GPS satellite, which can be calculated by the coordinates of the LEO and the GPS satellite.

According to the GPS observation, TEC_1 is expressed as below.

$$TEC_1 = \frac{f_1^2 f_2^2}{40.3(f_1^2 - f_2^2)} (P_1 - P_2) + Bias \quad (4)$$

where f_1 and f_2 are GPS satellite frequencies ($f_1 = 1575.42$ MHz, $f_2 = 1227.6$ MHz); P_1 and P_2 are measured pseudo ranges by dual-frequency GPS receivers. The *Bias* is the sum of the hardware error, multipath error and measurement noise error. In this paper, we use carrier

phase to smooth pseudo ranges with a Kalman filter algorithm to get more accurate smoothed pseudo ranges $P_{1,smooth}$ and $P_{2,smooth}$, then replace P_1 and P_2 with them in equation (4). The smoothing algorithm is described in Section 2.2.

Also assuming the “no bias” TEC as the function of latitude and longitude, and using Least Squares to estimate the *Bias* (details are explained in 2.2), then TEC_1 is generated (Gaussiran et al., 2004).

With the combination of equations (3) and (4), N_1 can be solved.

Now we need to correct N_1 . When GPS and LEO satellites move, the RO signals will pass every layers many times. Also the RO signal sampling interval is one second and every RO event continues for about three minutes. Therefore, we get many pairs of $(\tilde{N}_1, \tilde{l}_1)$. \tilde{N}_1 is layer 1 electron density acquired by every GPS observation and \tilde{l}_1 is the distant between LEO and GPS response to every observation. Through the Least Squares method, “less bias” N_1 is generated (Stranger et al., 1997).

After finishing N_1 calculation, we start to calculate layer 2 electron density N_2 by equation (2). That is as below.

$$N_2 = \frac{TEC_2 - 2 \cdot l_{1,2} \cdot N_1}{2 \cdot l_{2,2}} \quad (5)$$

The calculation steps are as above description.

Through this “peeling onion” method, all 10 layers electron densities are generated through one or more occultation events data.

When all the 10 layers electron densities N_i ($i = 1$ to 10) are calculated, in next step, we need to form an electron density function $N(h)$ with height as parameter.

According to the electron density empirical model, we consider an electron density mathematical model as below.

$$N(h) = A_0 + A_1 h + A_2 h^2 + A_3 h^3 + A_4 h^4 + A_5 h^5 \quad (6)$$

where:

$N(h)$ is electron density in h height.
 $A_0, A_1, A_2, A_3, A_4, A_5$ are coefficients.

Putting the solved N_i ($i=1$ to 10) and their corresponding height h_i into Equation (6), then get $A_0, A_1, A_2, A_3, A_4, A_5$ by Least Squares. Finally $N(h)$ is settled, which can be used as a TEC vertical distribution profile.

Vertical TEC (VTEC) response to latitude, longitude and time

The definition of VTEC in a space point $A(\theta, \lambda, h)$ is the total electron content along the ray path which is perpendicular to the ground and starting from ground to unlimited boundary with latitude θ and longitude λ . It is a 2 dimensional space function with latitude, longitude and time as variables, without relation to altitude.

In a space point $A(\theta_h, \lambda_h, h)$ where θ_h is latitude, λ_h is longitude and h is altitude, the following relationships are established:

$$\begin{aligned} \theta_h &= \sin^{-1}(\sin(\theta)\cos(p) + \cos(\theta)\sin(p)\cos(a)) \\ \lambda_h &= \lambda + \sin^{-1}(\sin(p)\sin(a)/\cos(\theta_h)) \\ p &= \frac{\pi}{2} - e - \sin^{-1}(a) \\ a &= \frac{R\cos(e)}{R+h} \end{aligned} \quad (7)$$

where e is the GPS satellite elevation angle, R is the Earth radius, other items refer to the detonations in **Figure 3**.

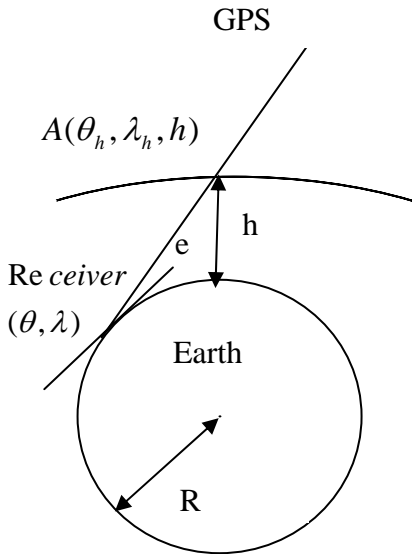


Figure 3: A space point in GPS signal Ray Path

At point $A(\theta_h, \lambda_h, h)$, its vertical TEC is as below.

$$VTEC(\theta_h, \lambda_h, t) = TEC_{slant} \cdot obliquity \quad (8)$$

with

$$TEC_{slant} = \frac{f_1^2 f_2^2}{40.3(f_1^2 - f_2^2)} (P_1(t) - P_2(t))$$

$$obliquity = \sqrt{1 - a^2}$$

$$a = \frac{R\cos(e)}{R+h}$$

where $VTEC(\theta_h, \lambda_h, t)$ is vertical TEC response to latitude θ_h and longitude λ_h . $P_1(t)$ and $P_2(t)$ are observed pseudo ranges in epoch t .

We use the smoothed pseudo ranges $P_{1smooth}(t)$ and $P_{2smooth}(t)$ to replace $P_1(t)$ and $P_2(t)$.

The algorithm uses carrier phase smoothed pseudo ranges by Kalman filter method as below.

$$\begin{aligned} P_{st}^- &= P_{s(t-1)}^+ + (L_t - L_{t-1}) \\ P_t^- &= P_{t-1}^- + q \\ k_t &= P_t^- (P_t^- + r)^{-1} \\ P_{st}^+ &= P_{st}^- + k_t (P_t - P_{st}^-) \\ P_t^+ &= (1 - k_t) P_t^- \end{aligned} \quad (9)$$

Where P_{st}^- is the estimated smoothed pseudo-range in epoch (t); $P_{s(t-1)}^+$ is the smoothed pseudo-range in epoch ($t-1$); L_t is the measured carrier phase in epoch (t); L_{t-1} is the measured carrier phase in epoch ($t-1$); P_t^- is the estimated error covariance matrix in epoch (t); P_{t-1}^+ is the error covariance matrix in epoch ($t-1$); q is the process noise covariance matrix, k_t is the Kalman gain in epoch (t), r is the measurement noise covariance matrix, P_{st}^+ is the smoothed pseudo range in epoch (t), P_t is the measured pseudo range in epoch (t), P_t^+ is the error covariance in epoch (t)

Figure 4 and **Figure 5** show the TEC comparison between the results from smoothed pseudo ranges and the raw pseudo-ranges.

In equation (8), VTEC includes bias because when calculating TEC_{slant} , all of the hardware error, multipath error and noise are not considered. Therefore, we need to correct VTEC.

We assume the “no bias” local VTEC (denoted as $F(\theta_h, \lambda_h)$) is a linear function of latitude θ_h and longitude λ_h .

$$F(\theta_h, \lambda_h) = A_0 + A_1\theta_h + A_2\lambda_h \quad (10)$$

Then $VTEC(\theta_h, \lambda_h, t)$ in equation (8) has another expression.

$$VTEC(\theta_h, \lambda_h, t) = Bias + F(\theta_h, \lambda_h) \quad (11)$$

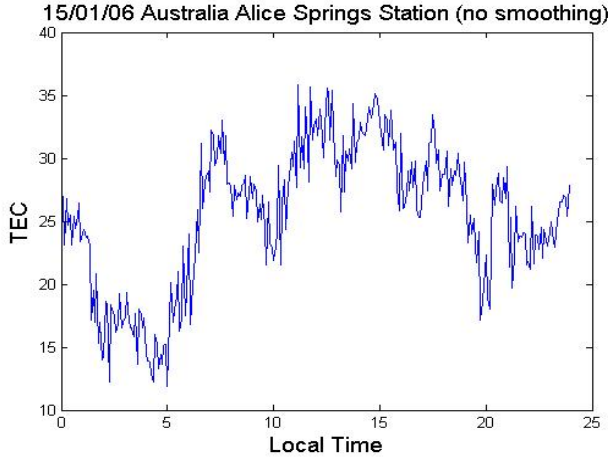


Figure 4: TEC daily variations by pseudo ranges
15/01/06 Australia Alice Station (smoothing)

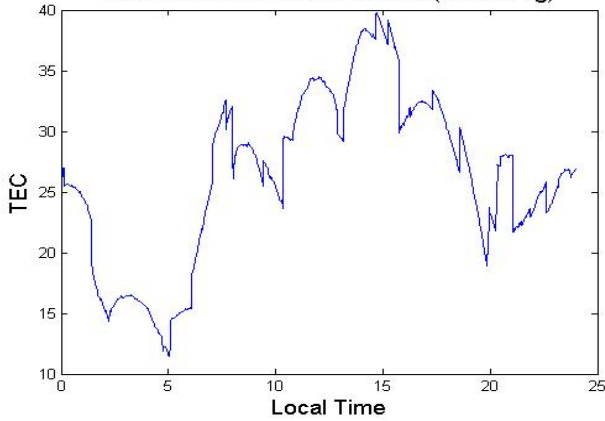


Figure 5: TEC daily variations by smoothed pseudo ranges

With Equations (10) and (11), then

$$\frac{f_1^2 f_2^2}{40.3(f_1^2 - f_2^2)} (P_{1,smooth} - P_{2,smooth}) \cdot Obliquity = (Bias, A_0, A_1, A_2) \begin{pmatrix} 1 \\ 1 \\ \theta_h \\ \lambda_h \end{pmatrix} \quad (12)$$

Using Kalman filter to solve the Bias, then the “less bias” VTEC is acquired as below:

$$VTEC(\theta_h, \lambda_h, t) - Bias \quad (13)$$

VTECH (vertical TEC with height variation) response to time, latitude, longitude and altitude

The VTECH is defined as a proportion VTEC starting from ground to a certain height. Then at a space point $A(\theta_h, \lambda_h, h)$ in epoch t, VTECH is expressed as below.

$$VTECH(\theta_h, \lambda_h, h, t) = VTEC(\theta_h, \lambda_h, t) \cdot p \quad (14)$$

Where

$$p = \frac{\int_{50}^h N(h) dh}{\int_{50}^{1000} N(h) dh}$$

$VTEC(\theta_h, \lambda_h, t)$ is “less bias” VTEC from equation (13). $N(h)$ is the vertical electron density profile from equation (6). Assuming the ionosphere starting from 50km above the surface of the Earth and extending to 1000km, and electron densities are zero below 50km and beyond 1000km.

According to the above equation, for a local area, the VTECH response to any space point in a certain epoch is calculated, and further more, a local TEC map is generated.

EXPERIMENTAL RESULTS

Figure 6 comes from COSMIC web site. It shows the trace of a RO event. It occurred from 2:34:15 to 2:37:09 in 16th October 2006 by GPS 31th satellite and LEO satellite COMIC L1. It crosses Australian continent sky and fits for generating TEC vertical distribution profile for Australia region.

Occultation map of atmPhs_C001.2006.289.02.34.G31_2007.1700.nc

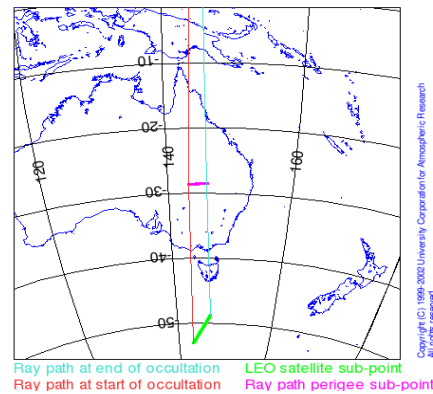


Figure 6: A RO Event over Australia Continent

Figure 7 shows the calculated TEC along the signal ray path between the LEO and GPS Satellite above Australia region. The TEC was calculated from the RO event data (shown in **Figure 6**).

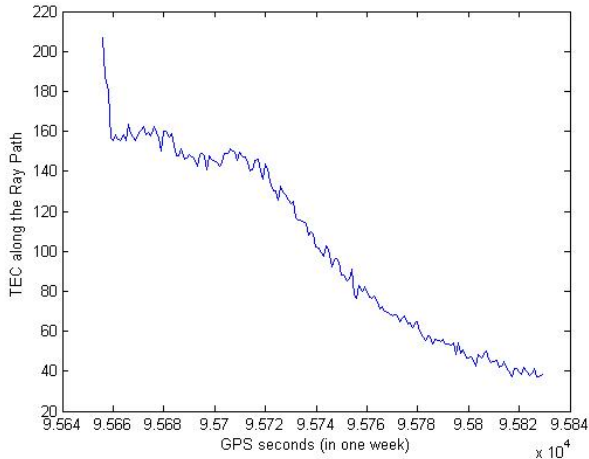


Figure 7: TEC along the ray path

Figure 8 shows the one hour (UT 2:00-2:59) average TEC value response to time, latitude and longitude in Australia region. We assume the TEC only concentrates on a thin layer located at 350km altitude above the surface of the Earth.

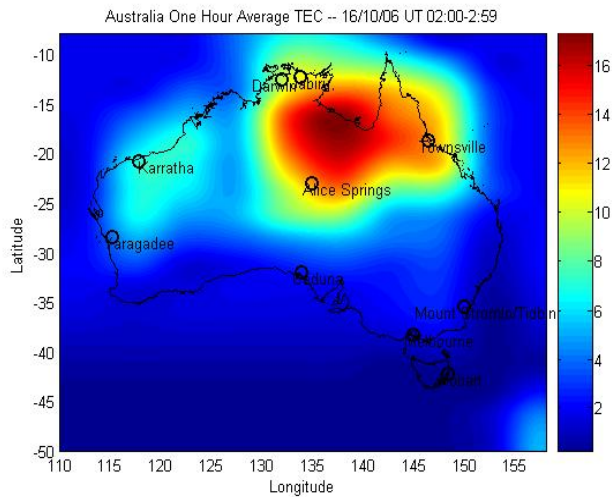


Figure 8: One Hour TEC Distribution Map

Figure 9 shows that one hourly (UT 2:00-2:59) average TEC value distribution with response to latitude and longitude in three different altitude. We can present above the TEC at any altitude. But for an illustration, we only exhibit the TEC distribution at three selected altitudes. This is a 3D TEC space distribution map with response to time,

latitude, longitude and altitude. In our future studies, more data sets will be processed to verify the reliability of the proposed procedure.

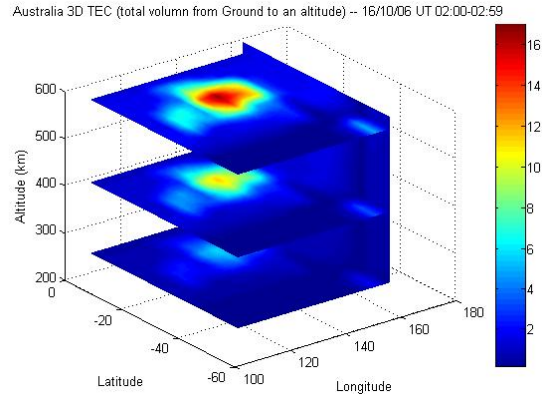


Figure 9: 3D TEC distribution in Australia region

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