GPS with Multiple Reference Stations: Surveying Scenarios in Metropolitan Areas

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ABSTRACT

The standard scenario for GPS surveying is to use two carrier phase-tracking GPS receivers, one located at a "reference" station with known coordinates and the second one at a station whose coordinates are to be determined. (It is also possible to determine a trajectory when the second receiver is in motion.) However, the cost of two GPS receivers, and associated software, is high and has precluded their use by many surveyors. One way of reducing the cost of GPS surveys is to propose that the reference station receiver be operated as a service by some government agency or private industry organisation. In this way the surveyor need only purchase a single GPS receiver, and does not need to deploy a second field party to operate the base receiver. Even further savings could be won if the data processing could be carried out by a third party, for example by submitting the recorded data to a WWW site that has access to the base receiver data. Despite the apparent advantage to the surveying industry of GPS reference stations operated for them as a service, there has not been a great interest in utilising them. The authors speculate that this may be because the perceived benefits of *single* reference stations are not significant enough. This paper discusses the advantages of continuously operating, GPS reference station networks that can service an entire metropolitan area, and deliver to users such benefits as increased performance, efficiency and flexibility compared with the standard single reference station scenario.

1. CARRIER PHASE-BASED GPS POSITIONING TODAY -- STATUS OF SINGLE REFERENCE STATION TECHNIQUES

Continuously operating GPS networks have been used for many years to address two categories of positioning applications. The first, and perhaps the best known application, is in support of geodetic objectives such as the determination of crustal motion on a variety of spatial scales. Currently, many hundreds of permanent GPS stations around the world are formally part of the global network known as the International GPS Service (IGS, 1999). The second class of applications are those that require real-time, differential GPS (DGPS) services over many hundreds of kilometres, to determine coordinates to accuracies of the order of a few metres. The DGPS corrections to pseudo-range data are determined at groups of permanent GPS stations which service wide continental or oceanic regions, using a positioning technique generally referred to as Wide Area DGPS (WADGPS) (see, e.g., Parkinson & Spilker, 1997), and then transmitted to users via geostationary communication satellites.

Similar WADGPS principles can also be used to significantly improve the accuracy of carrier phase-based static and kinematic GPS positioning (Rizos & Han, 1998). This paper presents the results of experiments in which several GPS reference station receivers located up to ???km apart have been used to support a typical static and kinematic survey in the Sydney urban area. The data processing algorithms have been developed to allow the processing of multi-reference station GPS data are briefly described. Two GPS surveying scenarios capable of centimetre-level accuracy, over baselines up to several tens of kilometres, are considered: (a) rapid static positioning using low-cost, single-frequency user receivers, and (b) highly efficient "on-the-fly" ambiguity resolution for medium-range, kinematic positioning. In both cases, the multiple reference station receivers have contributed to improved positioning performance. Before describing these experiments it is useful to review the principles of carrier phase-based static and kinematic GPS positioning.

1.1 The Evolution of Static GPS Surveying

The first application of the processing of carrier phase data was developed in the early 1980s, and led to the introduction of the technique of "GPS surveying". This was distinguished from the conventional "GPS navigation" operational scenario by virtue of: (a) the points being coordinated were stationary, (b) GPS data were collected over an "observation session", (c) the relative positioning mode of operation was used, (d) the measurements were made on the L-band carrier wave, and (e) the results were not required immediately (i.e., not in "real-time"). Since the mid-1980s the technique of static GPS surveying has been extensively used for the establishment of geodetic control and nowadays it is unthinkable that conventional terrestrial surveying technology would be used for this task. The main advantage of GPS was (and still is) that it permitted the determination of the position of one GPS receiver relative to another (reference) receiver *without the requirement for station intervisibility* . This characteristic of GPS far outweighs the most negative aspect of carrier phase-based GPS positioning, that is, the necessity to collect data over an observation session of up to several hours in length (or several days in the case of ultra precise geodetic GPS techniques). The adoption of the GPS technology by many surveyors was hindered by the issue of low "productivity" (defined in terms of number of surveyed points per day) and the high cost of the specialised (carrier phase tracking) GPS receivers and post-mission data processing software.

Since the early 1990s successive innovations have been introduced by the manufacturers. Nowadays carrier phase-based GPS positioning is possible even while the antenna is in motion (i.e. "kinematic positioning"), in real-time, with comparatively short observation spans (equivalent to "station occupation time" in the case of static surveys). All of these have made carrier phase-based techniques more attractive to the general class of survey applications, such as cadastral, hydrographic and engineering surveying. *What has been responsible for this improvement in performance?*

Phase measurements on the GPS signal are inherently *ambiguous* (that is, they are not a direct measurement of distance), the analysis of GPS phase data is a relatively complex operation. This is particularly the case for the algorithm which "resolves" the carrier phase ambiguities, one for each receiver-satellite pair, to their (theoretical) integer values. It was only with the development of sophisticated ambiguity resolution (AR) algorithms that it was possible to reduce the length of time required for static GPS surveys, for it is only by processing carrier phase data with resolved (or "fixed") ambiguities that centimetre level baseline accuracies can be assured (see Han & Rizos, 1997a, for a review of modern AR techniques).

The "stop & go" technique was one technique that was proposed in order to reduce the station occupation time (Remondi, 1988). It does require that the integer ambiguity values be resolved at an *initialisation* step, and then for the user receiver to maintain GPS signal tracking as it moves from one station to another (hence the resolved ambiguities remain valid for all subsequent positioning). Another GPS surveying technique commonly used nowadays is known as "rapid static" (also "quick static" or "fast static"), and has directly benefited from improvements to the AR algorithms. There is no operational difference compared with the original static GPS surveying technique except that the station occupation time has been reduced to the order of minutes (5-15 minutes is typical). However, this technique does require that the distance between the reference receiver and the user receiver be comparatively short (<10-15km). When distances between GPS receivers are longer, perhaps tens of kilometres (as in the case of geodetic control networks) to hundred of kilometres (as in the case of geodynamic surveys), observation session must also be lengthened. Observation session lengths for medium-to-long baseline determination may be hours to several days.

However, if we restrict the baseline lengths then with progressive improvements in GPS hardware even more dramatic reductions in "observation time for AR" have been achieved. Today's dual-frequency GPS receivers measure carrier phase on both the L1 and L2 carrier waves, as well as making accurate (decimetre level) pseudo-range measurements using the P-code PRN modulations on both L-band frequencies. It is this combination of dual-frequency carrier phase and precise pseudo-range data that now permit commercial "off-the-shelf" GPS systems to resolve ambiguities in a matter of tens of seconds, even while the receiver is moving (see §1.2). AR with only a single epoch of data has been demonstrated in university tests (Han, 1997a; 1997b).

Hence the progressive shortening of the observation session, *without sacrificing very much in the way of accuracy*, has made GPS more competitive with conventional terrestrial surveying technology. Nowadays GPS can provide centimetre accuracy results with less than a minute of data for AR (and even less in the case of "stop & go" if the ambiguities have already been resolved). *What are the "constraints" on this performance?* We can identify two: (a) the high cost of these top-of-the-line, dual-frequency GPS receivers, and (b) the restriction on the length of baseline, typically less than 10-15km. Both of these constraints to rapid static GPS surveying can be addressed in the multi-reference station receiver scenario.

1.2 The Evolution of Carrier Phase-Based Kinematic GPS Positioning

Carrier phase-based kinematic positioning is one of the most difficult engineering challenges for GPS system developers. Over the past half decade or so several developments have occurred which deliver high accuracy performance in "real-time" –- that is, in the field, immediately after making the measurements (but after the data from the reference receiver has been transmitted to the user receiver's computer for processing). Real-time positioning is even possible when the GPS receiver is in motion (with AR being carried out using a so-called "on-the-fly", or OTF, algorithm). These systems are commonly referred to as RTK systems ("real-time-kinematic"), and for the first time make *feasible* the use of GPS for time-critical applications such as machine control, GPS-guided earthworks/excavations, automated container port operations, etc. Although RTK systems represent the "state-of-the-art" in GPS technology, able to deliver centimetre level accuracy in real-time using a pair of GPS receivers, there are several conditions (or constraints) that must be fulfilled. These constraints may be so restrictive that they prevent the widespread adoption of precise GPS technology for both engineering surveys (a traditional application of carrier phase-based GPS techniques), as well as for new applications such as precise navigation in support of autonomous robot vehicle operation (identified by many as one of the new "frontier" applications of GPS).

If GPS signals were continuously tracked and loss-of-signal-lock never occurred, the integer ambiguities determined at the beginning of a survey would be valid for the whole period that GPS was being used, whether the antenna was stationary ("static" positioning) or moving ("kinematic" positioning). However, the GPS satellite signals can be shaded (e.g., due to buildings in "urban canyon" environments, or when the receiver passes under a bridge or through a tunnel), in which case the ambiguity values are "lost" and must be redetermined. *This process can take from several tens of seconds up to a few minutes with present GPS commercial-off-the-shelf (COTS) systems, but only when the reference-to-rover receiver distance is less than about 10-15km.* During this "re-initialisation" period centimetre accuracy positioning is not possible, and hence there is "dead" time until sufficient data has been collected to "resolve the ambiguities" (see Figure 1). If interruptions to the GPS signals occur repeatedly, then ambiguity re-initialisation is at the very least an irritation, and at worse a significant weakness of GPS COTS carrier phase-based systems. In addition, the longer the period of tracking required to ensure reliable on-the-fly AR (OTF-AR), the greater the risk that cycle slips will occur during this crucial (re-)initialisation period. These shortcomings are also present in any system based on data post-processing as well. Figure 1 also illustrates the situation when the OTF-AR algorithm is so optimised that it can operate on a single epoch of data, as in the UNSW OTF-AR algorithm (Han, 1997a; 1997b).

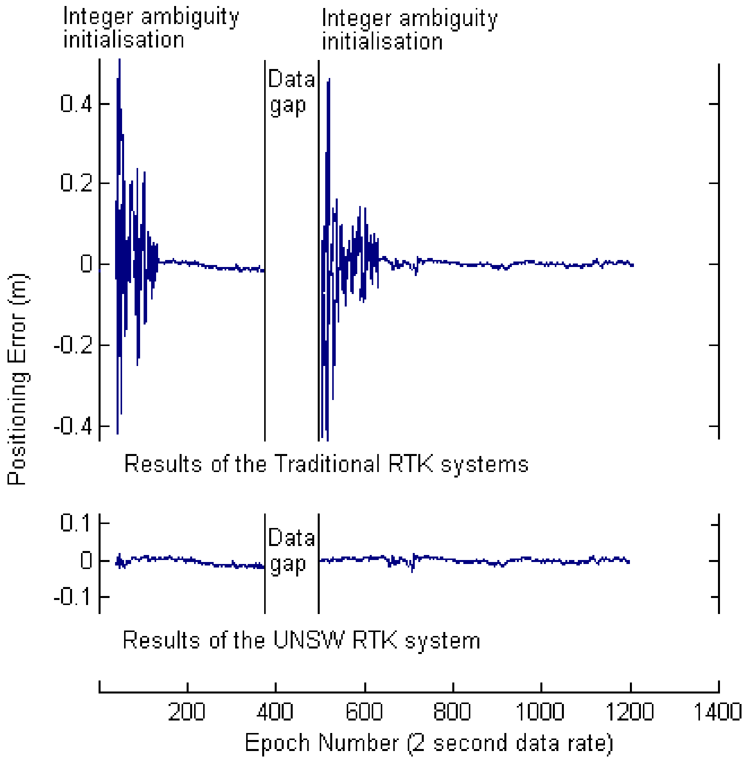

Figure 1. Comparison of RTK performance using standard OTF-AR techniques and the instantaneous OTF-AR technique used by the UNSW software.

The goal of all GPS manufacturers is to develop the ideal, real-time, precise GPS positioning system, able to deliver positioning results, on demand, in as easy and transparent a manner as is presently the case using pseudo-range-based differential GPS (DGPS) techniques, which typically deliver positioning accuracies in the range 1-10 metres. For example, the DGPS technique is robust, implemented in real-time via the transmission of pseudo-range correction data, and there is negligible delay in obtaining results. However, there are significant challenges for the developers of a similarly reliable "plug-and-play" positioning system that is capable of centimetre level accuracy:

(a) Residual biases or errors after double-differencing can only be neglected for AR purposes when the distance between two receivers is less than 10-15km. For medium-range or long-range precise GPS kinematic positioning, the distance-dependent biases, such as orbit bias, ionospheric delay and tropospheric delay, will become significant problems (Han, 1997b).

(b) Determining how long the observation span should be for reliable AR is a challenge for real-time GPS kinematic positioning. The longer the observation span is required, the longer the "dead" time during which precise positioning is not possible (see Figure 1). This can happen at the ambiguity initialisation step if the GPS survey is just starting, or at the ambiguity re-initialisation step if the GPS signals are blocked causing cycle slips or data interruptions.

(c) AR techniques typically require five or more visible satellites and expensive dual-frequency GPS instrumentation in which the application of geometric constraints and the combination of dual-frequency observations make AR easier (Han & Rizos, 1997a).

(d) Data latency is a challenge for many time-critical applications. The data latency is normally caused by the data transmission and the data processing, both of which cannot be avoided. Even if the data latency is only of the order of a few tenths of seconds, it may restrict many applications.

(e) Quality control of the kinematic GPS positioning results is a critical issue and is necessary during all steps: data collection, data processing and data transmission. However, the development of highly reliable quality control or validation criteria for AR is a significant challenge.

Over the last few years several important developments have occurred that appear to have overcome some of the constraints for carrier phase-based kinematic GPS positioning:

- Under certain conditions decimetre level positioning accuracy has been possible even when the baseline lengths have been from a few tens up to hundreds of kilometres in length (Han & Rizos, 1997b; Han et al., 1998).
- Reliable OTF-AR in the shortest period of time possible, even with just one measurement epoch, has been demonstrated (Han, 1997a; 1997b). Given very short periods of time-to-AR the notion of cycle slips, or having to re-initialise the ambiguities, has no meaning because so-called *instantaneous* OTF (IOTF) becomes the normal mode of kinematic positioning for *all epochs*.

The most significant algorithm improvements therefore have been in: (a) shortening the "time-to-AR" to just one epoch of data, and (b) overcoming the baseline length constraint. Together with advances in GPS hardware (principally the availability of dual-frequency receivers capable of making carrier phase and precise pseudo-range measurements on both the L1 and L2 signals), developments in fast ambiguity resolution algorithms and validation criteria procedures, improvements in the observation stochastic modelling and the application of careful quality control procedures, have generally been responsible for this increased level of AR performance. However, using a single reference GPS receiver still requires that the baseline lengths be less than about 10km, a significant constraint for many high precision kinematic positioning applications. Although carrier phase-based, *medium-range* GPS kinematic positioning has been reported for baselines several tens of kilometres in length (see, e.g., Wübbena et al., 1996; Han, 1997a; Han & Rizos, 1997b), such exceptional positioning performance requires the use of multiple reference stations in order to mitigate the effect of satellite orbit bias, as well as the ionospheric and tropospheric biases, multipath and observation noise.

1.3 Permanent, Continuously Operating GPS Reference Stations

Continuously operating GPS networks have been used for many years to address geodetic applications such as the determination of crustal motion on local, regional and continental scales. Many hundreds of permanent GPS stations around the world contribute data to the International GPS Service. Example of continental scale networks is the Australian Fiducial Network (AFN), comprising less than ten GPS receivers (AUSLIG, 1999). The inter-station distances are over one thousand kilometres. Networks have also been established to study local ground deformation due to tectonic faulting. Examples of such networks include the Southern California Integrated GPS Network (SCIGN) (Bock et al., 1997), and the Geographical Survey Institute's (GSI) network in Japan now consisting of almost 1000 permanent GPS stations (Sagiya et al., 1997). Such networks collect and archive carrier phase data on a daily basis. However, rarely do surveyors use this data to supplement their GPS surveys.

As noted earlier, considerable savings would be won if a surveyor were able to purchase only a single GPS receiver, and to use data collect by a third party at GPS reference station(s). The surveyor could collect data at the points of interest (in static or kinematic positioning mode), and then upon returning to the office could download the data from the nearest GPS reference station and perform the baseline computation(s). However, as discussed in §1.1 & 1.2, modern GPS techniques require the reference receiver be within 10 or so kilometres of the survey area. The establishment of a network of GPS reference receivers at a density to support all (or most) GPS surveys is hardly feasible. The GSI network has an average receiver spacing of 20-30km, and the data is now being made available to the surveying community. In some states of Germany GPS receivers are being established by survey authorities with similar spacings. Closer to home, the state of Victoria will also establish a GPS reference station network to support GPS surveys. However, it was envisaged that these networks would support single reference receiver baseline determination techniques, such as are implemented within commercial GPS data processing software. *These are less efficient than using multi-reference station techniques.*

2. MULTIPLE GPS REFERENCE STATION TECHNIQUES -- HOW IT CAN IMPROVE GPS SURVEYING PERFORMANCE

The advantages of carrier phase-based, multi-reference station GPS positioning techniques can be summarised as:

(a) Rapid static and kinematic GPS positioning techniques can be used over baselines up to a hundred kilometres in length.

(b) Instantaneous OTF-AR algorithms can be used for kinematic GPS positioning, at the same time ensuring high accuracy, availability and reliability for critical applications.

(c) Rapid static positioning is possible using low-cost, single-frequency GPS receivers, even over tens of kilometres.

Hence the question of "how to improve GPS surveying performance?" is answered in various ways. If the baseline length constraint is the crucial factor impacting on static or kinematic GPS, then overcoming this constraint through using multi-reference receiver techniques results in significant performance improvements. If "improving performance" is related to lowering receiver costs, then a multi-reference receiver network would allow the use of single-frequency GPS receivers instead of expensive dual-frequency receivers for static positioning. If "time-to-AR" is the critical performance indicator, then a multi-reference receiver system would contribute to faster AR (including single epoch AR) as well as enhance the reliability of AR, even for baselines several tens of kilometres in length. However, as a user, the implementation of a GPS positioning technique based on data from multiple reference stations is much more complicated than the standard single reference receiver scenario.

2.1 Implementing Multi-Reference GPS Receiver Techniques

In order to win the performance improvements mentioned above more than two GPS receivers would need to be used to support the survey. Rarely would the benefits of improved positioning performance be great enough for a single user that they would be compelled to establish and operate their own reference receiver network. The authors have therefore assumed that a multi-reference station *service* would be provided by a government agency or private organisation. This may be a "fee-for-service" or the reference receiver data may be provided free of charge. Furthermore, in order to justify such an infrastructure, the authors have proposed that the reference receiver network would be established in order to service the GPS survey needs of a large metropolitan area such as that centred on Sydney or Melbourne. According to such a scenario three or four GPS reference receivers would be deployed on the outskirts of the city so that they would form a polygon surrounding the area in which GPS surveys would be carried out.

it is envisaged that some or all of the following three types of services could be offered across a metropolitan area:

(a) The "minimalist" option is for the user to simply download (via the Internet or a Bulletin Board Service) the necessary data files from the reference station network. No modification is needed to present, government-run GPS services such as GSI's or AUSLIG's. The user, however, requires the appropriate post-mission data processing software to do the multi-station processing. This is not generally available from commercial sources.

(b) One of the more innovative implementations of multi-reference station processing is <u>not</u> to require the surveyor to download *any* GPS reference receiver data. Rather, the user logs the GPS data locally and then submits (or uploads) the data file to a central server for processing via an Internet Web page (the web browser software necessary for this could be installed on a mobile phone such as Nokia's 9110??? or a palmtop PC "organiser"). The implementation of such a service requires the establishment of the appropriate GPS reference station data links to the central server, a database management system and the web-based baseline processing "engine". This service has the additional advantage in that it obviates the need for surveyors to maintain their own PC-based GPS data processing software. Furthermore, this service is ideally suited for the more sophisticated data processing necessary when using <u>all</u> the data from the reference station network, and would be expected to be offered on a fee-for-service basis.

(c) The other type of service would be for real-time users. Unfortunately this implementation is more complex for a number of reasons. Firstly, the distance(s) from reference receiver(s) to user may be several tens of kilometres, resulting in communication link issues becoming critical. Secondly, because more than one GPS reference receiver is used, a network-generated correction message must be transmitted. This is not unlike a Wide Area DGPS (WADGPS) scenario, except that carrier phase data is used for positioning instead of pseudo-ranges. In WADGPS services a satcom link is typically used, and a special decoder is necessary (for which a subscription fee is paid) because the correction message that must be input to the user's RTK receiver is *location dependent*. Such services may not be too far off. For example, Fugro will shortly commence testing of its HADS (High Accuracy DGPS Service), which the authors suspect is a form of carrier phase-based, real-time, multi-reference station positioning technique.

Before describing an experiment that simulates static and kinematic implementations of multi-reference station positioning, the mathematical basis for the technique is briefly described. For more details the reader is referred to Han (1997a).

2.2 The Observation Model

Consider the scenario in Figure 2. Three or more permanent, dual-frequency receivers surround a network of benchmarks that are to be surveyed by a GPS receiver (single-frequency or dual-frequency). Processing of the data collected at the reference stations is necessary in order to determine the double-differenced integer ambiguities between them.

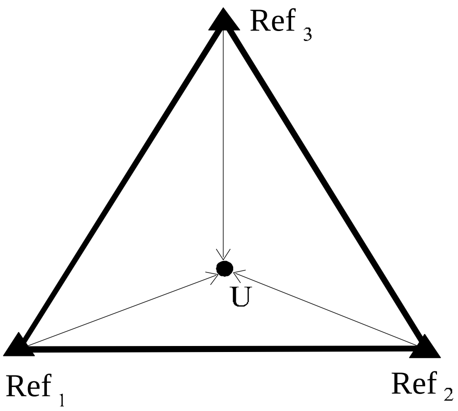

Figure 2. Configuration of the GPS reference stations (triangles) and the user receiver (circle).

The single-differenced carrier phase observation can be written as (Han, 1997a):

$$\Delta f_i = \Delta r_i + \Delta dr_i - c \cdot \Delta dT_i + l \cdot \Delta N_i - \Delta d_{ion,i} + \Delta d_{trop,i} + \Delta d^{f}_{mp,i} + e_{\Delta f_i} \qquad (1)$$

where $\Delta(\cdot)_i = (\cdot)_u - (\cdot)_i$; *i* indicates the reference station *i*, and *u* the user station; f: the carrier phase observation in units of metres; $r := \|X^s - X\|$, X^s is the satellite position vector, X is the station position vector; dr : is the effect of satellite orbit errors; dT: the receiver clock error with respect to GPS time; d_{ion}: the ionospheric delay; d_{trop}: the tropospheric delay after model correction; d^f_{mp}: the multipath on the carrier phase; e_f: the carrier phase observation noise; l: the wavelength of the carrier phase; and, N: the integer ambiguity.

A set of parameters a_i can be determined, based on the conditions given in Han & Rizos (1997b):

$$\sum_{i=1}^{3} a_i = 1 \qquad \text{and} \qquad \sum_{i=1}^{3} a_i\left(\vec{X}_u - \vec{X}_i\right) = 0 \tag{2}$$

where $\vec{X}_u$ and $\vec{X}_i$ (i=1,2,3) are the position in the Gauss plane coordinate system. If more than 3 reference stations are used, $\sum_{i=1}^{3} a_i = 1$ should be introduced in order to reduce the observation noise (see equation (8)). The linear combination of the single-differenced observations can be formed:

$$\begin{aligned}\sum_{i=1}^{3} a_i\cdot\Delta f_i = {} & \sum_{i=1}^{3} a_i\cdot\Delta r_i + \sum_{i=1}^{3} a_i\cdot\Delta dr_i - c\cdot\sum_{i=1}^{3} a_i\cdot\Delta dT_i + l\cdot\sum_{i=1}^{3} a_i\cdot\Delta N_i - \\ & - \sum_{i=1}^{3} a_i\cdot\Delta d_{ion,i} + \sum_{i=1}^{3} a_i\cdot\Delta d_{trop,i} + \sum_{i=1}^{3} a_i\cdot\Delta d^f_{mp,i} + e_{\sum_{i=1}^{3} a_i\cdot\Delta f_i}\end{aligned} \tag{3}$$

The orbit bias term has been shown to be (Han, 1997a):

$$\sum_{i=1}^{3} a_i\cdot\Delta dr_i \approx 0 \tag{4}$$

An epoch-by-epoch and satellite-by-satellite ionospheric model can be applied to estimate the single-differenced ionospheric delay relative to reference receiver 3. It has been shown that the interpolation of the single-differenced TEC (Total Electron Content) between receivers, or double-differenced TEC based on an ionosphere layer, can be simplified to that of an interpolation dependent on the user receiver's position in the Gauss plane coordinate system (ibid, 1997a). The ionospheric delay term can be represented as (ibid, 1997a):

$$\boxed{\sum_{i=1}^{3} a_i\cdot\Delta d_{ion,i} = d_{ion,u} - d_{ion,3} - \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}^T \begin{bmatrix} d_{ion,1} - d_{ion,3} \\ d_{ion,1} - d_{ion,3} \end{bmatrix}} = 0 \tag{5}$$

When the distance between the reference stations increases, the residual error will become greater due to residual effects arising from this ionospheric delay interpolation.

If it can be assumed that the tropospheric delay can be interpolated from the residual tropospheric delay at the reference stations, the residual tropospheric delay can be represented as:

$$\sum_{i=1}^{3} a_i\cdot\Delta d_{trop,i} = d_{trop,u} - d_{trop,3} - \begin{bmatrix} x_u & y_u \end{bmatrix} \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix}^{-1} \begin{bmatrix} d_{trop,1} - d_{trop,3} \\ d_{trop,1} - d_{trop,3} \end{bmatrix} \tag{6}$$

where (x_u, y_u), (x_1, y_1) and (x_2, y_2) are North and East coordinates relative to reference station 3 in the Gauss plane coordinate system. How closely it should be to zero depends on the spatial correlation of tropospheric delay. The residual tropospheric delay is mostly contributed to by the wet component of the troposphere, which shows strong variation with height, time and location. It can only be expected that the term $\sum_{i=1}^{3} a_i \cdot \Delta d_{trop,i}$ will be mitigated to some extent as a function of the distance between stations. The multipath term can be rewritten as:

$$\sum_{i=1}^{3} a_i \cdot \Delta d_{mp,i}^{f} = d_{mp,u}^{f} - \sum_{i=1}^{3} a_i \cdot d_{mp,i}^{f} \tag{7}$$

The last term $\sum_{i=1}^{3} a_i \cdot d_{mp,i}^{f}$ on the right hand side of equation (7) is the weighted mean value of the multipath values at the three reference receivers for this satellite. Due to the random nature of multipath at the different receivers, the weighted mean value will be significantly reduced if all a_i (i=1,2,3) are positive and less than 1.

The standard deviation of the one-way carrier phase observation can be considered as being a function of the elevation angle. Because all stations are located within a region of about 100km radius, the elevation of a satellite is approximately the same. The standard deviation of the one-way carrier phase observation can also be approximated by s^j and then the standard deviation of the linear combination of single-differenced observations $e_{\sum_{i=1}^{3} a_i \cdot \Delta f_i}$ can be expressed as:

$$s_{\sum_{i=1}^{3} a_i \cdot \Delta f_i} = \sqrt{1 + a_1^2 + a_2^2 + a_2^2} \cdot s^j \tag{8}$$

Compared to the standard deviation of the single-differenced carrier phase observation $\sqrt{2} \cdot s^j$, the standard deviation will become smaller if the user receiver is located within the triangle formed by the reference stations (Han, 1997b).

The single-differenced carrier phase observation functional model can be simplified to:

$$\sum_{i=1}^{3} a_i \cdot \Delta f_i = \sum_{i=1}^{3} a_i \cdot \Delta r_i - c \cdot \sum_{i=1}^{3} a_i \cdot \Delta dT_i + l \cdot \sum_{i=1}^{3} a_i \cdot \Delta N_i + e_{\sum_{i=1}^{3} a_i \cdot \Delta f_i} \tag{9}$$

Equation (9) can be written as:

$$\begin{aligned} \Delta f_{u,3} - \left[a_1 \cdot \Delta f_{1,3} + a_2 \cdot \Delta f_{2,3}\right] = {} & \Delta r_{u,3} - \left[a_1 \cdot \Delta r_{1,3} + a_2 \cdot \Delta r_{2,3}\right] - c \cdot \sum_{i=1}^{3} a_i \cdot \Delta dT_i + \\ & + l \cdot \Delta N_{u,3} - \left[a_1 \cdot \Delta N_{1,3} + a_2 \cdot \Delta N_{2,3}\right] + e_{\sum_{i=1}^{3} a_i \cdot \Delta f_i} \end{aligned} \tag{10}$$

Define the residual vectors:

$$V_{1,3} = \nabla\Delta f_{1,3} - \nabla\Delta N_{1,3} - \nabla\Delta r_{1,3} \tag{11}$$

$$V_{2,3} = \nabla\Delta f_{2,3} - \nabla\Delta N_{2,3} - \nabla\Delta r_{2,3} \tag{12}$$

The double-differenced observation model can then be written as:

$$\nabla\Delta f_{u,3} - \left[a_1 \cdot V_{1,3} + a_2 \cdot V_{2,3}\right] = \nabla\Delta r_{u,3} + l \cdot \nabla\Delta N_{u,3} + e_{\sum_{i=1}^{3} a_i \cdot \nabla\Delta f_i} \tag{13}$$

Using the known coordinates and known integer ambiguities, the correction vectors $V_{1,3}^{L1}$, $V_{1,3}^{L2}$ can be computed and made available for baseline processing. In the post-mission implementation, the correction terms $\left[a_1 \cdot V_{1,3} + a_2 \cdot V_{2,3}\right]$ in equation (13) for the L1 and L2 carrier phase observations are computed using a modified version of the Bernese GPS data processing software package. These are then input to the baseline determination software (the baseline is defined as being between reference receiver 3 and the user receiver). For a real-time implementation, the correction vectors, together with the carrier phase and pseudo-range data at reference station 3, would have to be transmitted to the user receiver *if its coordinates are approximately known*. A more general form would be to transmit to *all* users the necessary data for a user's receiver to compute the necessary correction terms for that particular location. ???Shaowei can you put a sentence here explaining what form the correction term would take???

3. EXPERIMENTAL RESULTS

The multi-reference station data processing methodology described above has been tested in several experiments across the Sydney area. Two user scenarios were investigated to confirm whether centimetre level positioning accuracy, over baselines up to several tens of kilometres, is possible: (a) rapid static positioning using single-frequency user GPS receivers, and (b) single epoch "on-the-fly" ambiguity resolution for medium-range, kinematic positioning. At the time of writing of this paper only the results from the first experiment were available.

3.1 Data Acquisition

??Chen & Chon please write up experiment in a similar format to that of an earlier test:

The experiment was carried out on 14 December 1996, using four ???? GPS receivers as the reference network (Figure 3). One ..... The other reference station was ...., the third ... and the fourth ..... The pattern of tracking is indicated in Figure 4. In the case of the kinematic experiment the mobile receiver was mounted on a car and the experiment started at the side of the M3 Freeway, ??km, ??km, ??km and ???km distant from the ?? receiver, the ??? receiver, the ??? receiver and the ??? receiver, respectively. After about ?? minutes of static occupation, the roving receiver started to move along the M3 Freeway, and then back to nearly the same point as the start point. The data rate was 1 Hz and a total of ??? epochs were used. The number of observed satellites was ???.

The coordinates of the reference stations were determined using the traditional long-range static positioning procedure and are referenced to the known coordinates of ????. After the integer ambiguities for the L1 and L2 carrier phase observations are resolved, the ambiguity-fixed solution is determined using the ionosphere-free phase combination and the IGS precise ephemeris (IGS, 1999). As a result, the locations of the reference stations can be considered as known.

Assume that the ?? station is reference station 3, and ?? and ?? are reference stations 1 and 2, respectively (reference station 4 was not used because .....). The correction terms $\left[a_1 \cdot V_{1,3}^{L1} + a_2 \cdot V_{2,3}^{L1}\right]$ and $\left[a_1 \cdot V_{1,3}^{L2} + a_2 \cdot V_{2,3}^{L2}\right]$ for the L1 and L2 carrier phase observations are plotted in Figures 5a, 5b, ???, for satellite PRNs ?? ?? ?? (PRN ?? is the reference satellite). In a similar way, the correction terms $\left[a_1 \cdot V_{1,3}^{R1} + a_2 \cdot V_{2,3}^{R1}\right]$ and $\left[a_1 \cdot V_{1,3}^{R2} + a_2 \cdot V_{2,3}^{R2}\right]$ can also be determined.

The static experiment involved ....

Figure 3

Figure 4

Figure 5???

3.2 Summary of Results

??Who will write this??

4. CONCLUDING REAMRKS

A scenario has been described in which a network of GPS reference stations could service GPS surveyors across a metropolitan area of the order of 50 x 50km or more. Multi-reference station positioning techniques can be used to improve the performance of high precision static and kinematic positioning. The methodology requires that the reference station data be first processed in order to generate correction terms. These correction terms, when applied to standard baseline determination software, mitigates the effects of satellite orbit error, as well as the ionospheric and tropospheric biases, multipath and observation noise. This makes possible single epoch OTF ambiguity resolution, even when the distance to the reference receiver(s) is greater than ??km. In addition, because the network-generated correction terms account for the residual ionospheric delay, high accuracy baseline results can be obtained even when using single-frequency user receivers.

Various scenarios for establishing a multi-reference station service have been described in this paper. The results presented here are very preliminary, and firm conclusions cannot be drawn. Further testing of the algorithms and procedures, for a variety of positioning applications, will be carried out in the coming year, in Sydney (using temporary UNSW GPS reference receivers), in Japan (using the permanent GSI reference receiver network) and in Singapore (where a multi-reference station test facility has recently been jointly established by the authors and their Singaporean colleagues).

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Chris Rizos holds a B.Surv. and Ph.D., both obtained from The University of New South Wales. He has been an academic staff member of the School of Surveying (renamed the School of Geomatic Engineering in 1994) since 1987, and is presently an Associate Professor. Chris has published over 100 papers, as well as having authored and co-authored several books relating to GPS and positioning technologies. He is leader of the Satellite Navigation and Positioning (SNAP) group at UNSW, specialising in precise static and kinematic applications of the GPS technology. SNAP is considered the premier academic GPS R&D group in Australia.

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