

Regional-Scale Multiple Reference Stations for Carrier Phase-Based GPS Positioning: A Correction Generation Algorithm

Chris Rizos, Shaowei Han and Horng-Yue Chen

School of Geomatic Engineering The University of New South Wales Sydney NSW2052, Australia

Continuously operating GPS networks have been used for many years in support of: (a) geodetic goals such as the determination of crustal motion on a variety of spatial scales, and (b) to provide pseudo-range corrections for Wide Area DGPS (WADGPS) implementations. Recently, regional-scale GPS permanent networks have been developed for multi-functional uses, including to support centimetre-accuracy, medium-range, carrier phase-based GPS positioning for surveying or precise navigation applications. In such an implementation the generation of carrier phase correction messages in a manner analogous to WADGPS requires that the integer ambiguities between GPS reference stations be fixed in real-time. Although the ambiguities could be resolved at the beginning of operation the challenge remains: how to recover an integer ambiguity if a cycle slip or data gap occurs, or if a new satellite rises? In this paper, the linear data combination algorithm that has been used is described, and the issue of "ambiguity recovery" techniques for data correction generation purposes is addressed. Three strategies are suggested: (1) using an "ambiguity recovery" technique if the data gap is shorter than a minute or so; (2) re-determining the integer ambiguities using an ionospheric correction derived from the tracking to other satellites when a new satellite rises, or after a long period of data loss, and; (3) re-determining the integer ambiguities with the aid of data corrections generated on previous days. Several 7-day continuous data sets were used for algorithm testing. The corrections can be generated from multiple reference stations in the post-processing mode and then used for comparison purposes with the simulated real-time processing mode using the proposed algorithm. Results confirm that the proposed algorithm can provide reliable carrier phase data corrections for centimetre-accuracy, real-time GPS positioning.

1. Introduction

Permanent GPS reference stations are being increasingly established all over the world. Regional-scale GPS permanent networks are now being deployed to support a multitude of applications:

- high precision, real-time GPS kinematic positioning over medium-range baselines.,
- rapid static positioning system using low-cost GPS receivers,
- machine control applications, e.g. precise farming, and
- deformation monitoring, e.g. volcano and structural monitoring.

In order to operate a regional-scale GPS network a number of the challenges must be addressed. One of them is how to generate carrier phase 'corrections' which could reduce distance-dependent errors such as orbit bias, ionosphere and troposphere delays, as well as station-dependent errors, e.g. multipath, measurement noise, etc.? Another is how to compute 'correction terms' in real-time, e.g. implementing real-time ambiguity resolution for reference stations over regional-scale areas? How to format and deliver the 'corrections' for different user applications? And so on.

A linear combination model for such network-based positioning has been proposed by Han (1997) and Han & Rizos (1997), which can account for orbit bias and ionospheric delay, as well as mitigate tropospheric delay, multipath and measurement noise across networks many tens of kilometres across. The procedure requires that the

data from the GPS reference station network be used to derive corrections to the double-differenced carrier phase data formed between any user receiver (located within the area bounded by the network) and a single reference receiver. Both static and kinematic positioning modes can be thus supported.

Li & Gao (1998), Odijk (1999) and Schaer et al. (1999) have studied the impact of atmospheric delay errors across regional-scale GPS networks, and have attempted to resolve the ambiguities in real time. However, network-based, real-time, carrier phase-based GPS positioning requires that the carrier phase 'corrections' generated from the reference station network data must be broadcast to users. To ensure service availability, all the integer ambiguities associated with the network processing must be fixed 'instantaneously'. In this paper, a correction generation algorithm suitable for the regional-scale GPS networks is proposed. The characteristics of the double-differenced ionospheric delay and tropospheric delay are discussed. Finally, the results for medium-range, single-frequency positioning will be presented to demonstrate the utility of network-based positioning.

2. Linear combination model

The single-differenced observations from a user receiver to multiple reference receivers are formed as:

$$\begin{aligned} \Delta \mathbf{f}_i &= \Delta \mathbf{r}_i + \Delta d\mathbf{r}_i - c \cdot \Delta dT_i + \mathbf{I} \cdot \Delta N_i \\ &- \Delta d_{ion,i} + \Delta d_{trop,i} + \Delta d_{mp,i}^f + \mathbf{e}_{\Delta \mathbf{f}_i} \end{aligned} \quad (1)$$

where $\Delta(\cdot) = (\cdot)_u - (\cdot)_i$; i : indicates the reference station i , and u the user station. Based on the conditions given in Han & Rizos (1996), a set of parameters can be computed:

$$\sum_{i=1}^3 \mathbf{a}_i = 1 \quad (2)$$

$$\sum_{i=1}^3 \alpha_i (\bar{X}_u - \bar{X}_i) = 0 \quad (3)$$

$$\sum_{i=1}^3 \mathbf{a}_i^2 = \min \quad (4)$$

where \bar{X}_u and \bar{X}_i are the coordinates of the user receiver and reference receiver i in the Gaussian plane coordinate system.

The linear combination of the single-differenced observations can be written as:

$$\begin{aligned} \sum_{i=1}^3 \mathbf{a}_i \cdot \Delta \mathbf{f}_i &= \sum_{i=1}^3 \mathbf{a}_i \cdot \Delta \mathbf{r}_i + \sum_{i=1}^3 \mathbf{a}_i \cdot \Delta d \mathbf{r}_i \\ &\quad - c \cdot \sum_{i=1}^3 \mathbf{a}_i \cdot \Delta d T_i + \mathbf{I} \cdot \sum_{i=1}^3 \mathbf{a}_i \cdot \Delta N_i \\ &\quad - \sum_{i=1}^3 \mathbf{a}_i \cdot \Delta d_{\text{ion}i} + \sum_{i=1}^3 \mathbf{a}_i \cdot \Delta d_{\text{trop}i} \end{aligned} \quad (5)$$

2.1 Features of linear combination technique

The linear combination technique proposed by Han & Rizos (1997) has several attractive features. First of all the orbit bias is taken into account, while the ionospheric delay is removed using an epoch-by-epoch and satellite-by-satellite ionosphere model. The tropospheric delay is reduced by interpolation based on the North and East coordinates of the reference and user receivers after applying an a priori troposphere model correction. The multipath at the reference receivers is mitigated by weighted averaging. The observation noise is mitigated by weighted averaging.

2.2 Correction term generation

Figure 1 illustrates the geometric relationship between the GPS reference stations and the user stations.

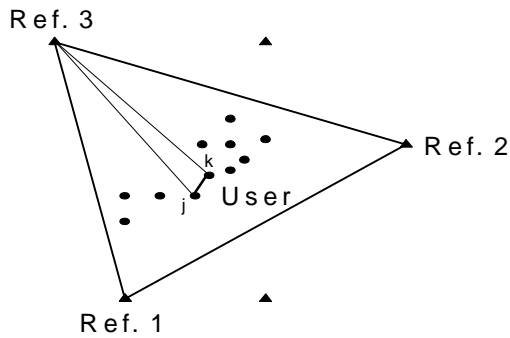


Figure 1. The triangles denote the GPS reference stations, and the dots denote the user stations.

Define the residual vectors:

$$\begin{aligned} V_{1,3} &= \nabla \Delta \mathbf{f}_{1,3} - \nabla \Delta N_{1,3} - \nabla \Delta \mathbf{r}_{1,3} \\ V_{2,3} &= \nabla \Delta \mathbf{f}_{2,3} - \nabla \Delta N_{2,3} - \nabla \Delta \mathbf{r}_{2,3} \end{aligned} \quad (6)$$

The double-differenced observation model can be written as:

$$\begin{aligned} \nabla \Delta \mathbf{f}_{u,3} - [\mathbf{a}_1 \cdot V_{1,3} + \mathbf{a}_2 \cdot V_{2,3}] \\ = \Delta \nabla \mathbf{r}_{u,3} + \mathbf{I} \cdot \nabla \Delta N_{u,3} + \mathbf{e}_{\sum_{i=1}^3 \mathbf{a}_i \cdot \nabla \Delta \mathbf{f}_i} \end{aligned} \quad (7)$$

Where the subscripts 1, 2, 3 represent the reference stations, and u denotes the user stations.

Using the known coordinates and known integer ambiguities, the correction vectors $V_{1,3}$, $V_{2,3}$ can be computed. Therefore, for real-time applications, the correction vectors, together with the carrier phase and pseudo-range data from reference station 3, must be sent to any user receiver.

3. Real-time ambiguity resolution

For medium-range, real-time, high accuracy applications, the ambiguities associated with the reference stations need to be fixed (i.e. resolved) *all of the time*. Then the correction term can be generated. When the reference system is set up, the integer ambiguities can be initialised. If a cycle slip or data gap of the order of a few minutes occurs, an ‘ambiguity recovery’ technique can be used (Han 1997). If after a long data gap, or when a new satellite rises, the relevant ambiguity will have to be estimated and resolved. This is quite a challenge due to the presence of residual (double-differenced) ionospheric and tropospheric delay over medium length baselines (defined here as being from several tens of kilometres to about 100km).

3.1 Ionospheric delay

The ionospheric delay exhibits some similarity between successive days. In addition, the ionospheric delay information derived from other tracked satellites can also help estimate the ionospheric delay for a newly risen satellite. Both of these types of ionospheric delay information can be used to generate a ‘synthetic’ ionospheric delay to assist ambiguity resolution for the reference station network processing.

The double-differenced ionospheric delay can be computed as:

$$\begin{aligned} \Delta \nabla I &= \begin{pmatrix} -f_2^2 \\ f_1^2 - f_2^2 \end{pmatrix} [(\Delta \nabla L_1 - \Delta \nabla N_1) \mathbf{I}_1] \\ &\quad - \begin{pmatrix} -f_2^2 \\ f_1^2 - f_2^2 \end{pmatrix} [(\Delta \nabla L_2 - \Delta \nabla N_2) \mathbf{I}_2] \end{aligned} \quad (8)$$

where L_1, L_2 are the carrier phase observations
 f_1, f_2 are the associated frequencies
 N_1, N_2 are the integer ambiguities for L_1, L_2
 λ_1, λ_2 are the wavelengths

If the integer ambiguity can be fixed, then the double-differenced ionospheric delay can be obtained using equation (8). As an example, Figure 2 displays the double-differenced ionospheric delay on satellite number 24 and 04 for two successive days.

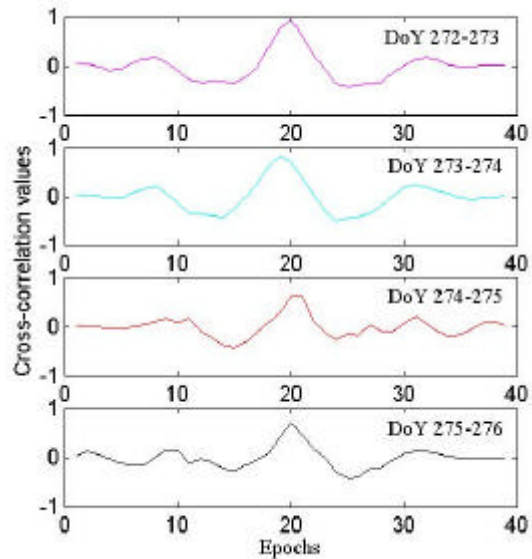
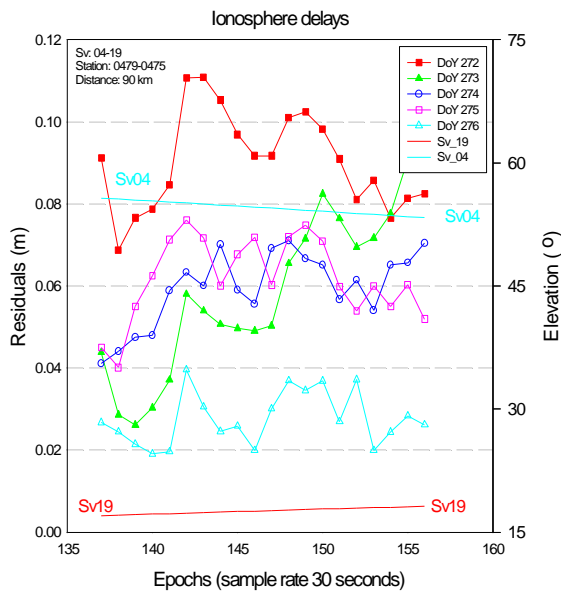
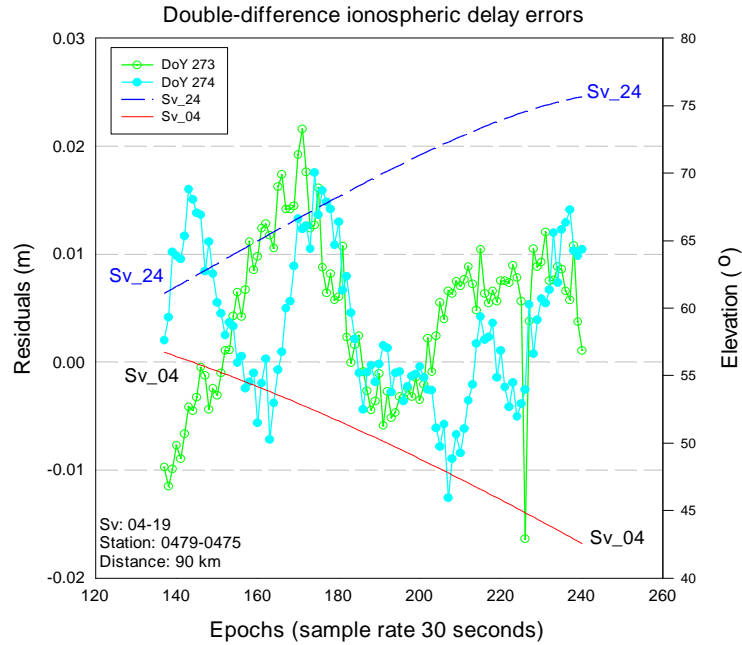


Figure 2. Double-differenced ionospheric delay in Kyushu, Japan, 1997.

Figure 3. Ionosphere delay similarity and cross-correlation value for 5 successive days (90km baseline).

The cross-correlation value in the case of a low elevation satellite (newly risen satellite Sv_19) from an analysis of the double-differenced ionospheric delay for 5 successive days at the stations 0479-0475 (90km baseline) is shown in Figure 3. The y-axis is the cross-correlation value and the x-axis is the epoch number. There are 20 epochs of the newly arisen satellite considered in this

cross-correlation test. The normalised cross-correlation value is 0.78.

3.2 Tropospheric delay

The tropospheric delay also has some similarity between successive days, and the tropospheric delay of the currently tracked satellites could be estimated by:

$$\Delta \nabla T_{trop} = \left(\frac{f_1^2}{f_1^2 - f_2^2} \right) ((\Delta \nabla L_1 - \Delta \nabla N_1) \mathbf{I}_1) - \left(\frac{f_1^2}{f_1^2 - f_2^2} \right) (\Delta \nabla L_2 - \Delta \nabla N_2) \mathbf{I}_2 + \Delta \nabla \mathbf{r} \quad (9)$$

The tropospheric delay for a newly risen satellite could then be estimated using both information sources. Figure 4 displays the double-differenced tropospheric delay for satellite number 10 and 04 on successive days, using equation (9).

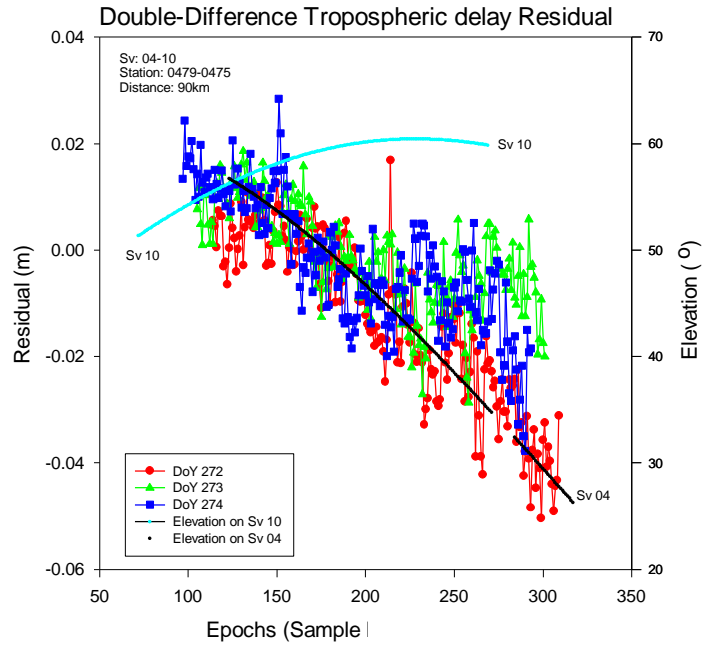


Figure 4. Double-differenced tropospheric delay residuals for satellite number 10 and 04 on successive days, using equation (9).

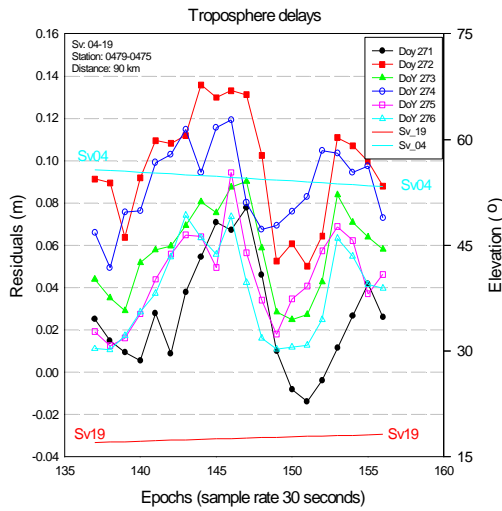


Figure 5. Tropospheric delay similarity and correlation for satellites Sv 04 and Sv 19 across different days of year.

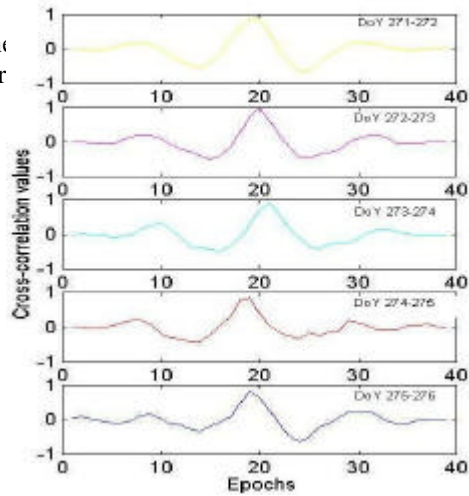


Figure 6. Cross-correlation values for tropospheric delays of satellites Sv 04 and Sv 19 across different days of year.

(90km baseline)

Figure 5 shows the tropospheric delay similarity and cross-correlation values of the double-differenced tropospheric delay between successive days (for a 90km baseline). The normalised cross-correlation values are at the 0.9 level.

Using the characteristic that the double-differenced ionospheric and tropospheric biases have a high correlation between successive days (as illustrated in Figures 3 and 5), when a new satellite rises a linear polynomial function model (based on data from successive days and previous epochs) can be determined, and a double-differenced ionospheric and tropospheric delay value for the new satellite can be estimated.

4. Experimental results

The experiment had two main objectives. The first was to study the ability of the polynomial function model (using data from successive days and previous epochs) to estimate the atmospheric delay when a new satellite rises or after a long data gap occurs. The second objective was to test the performance of the linear combination observation model for medium-range GPS rapid static positioning assuming the user receiver is a single-frequency instrument. Figure 6 displays one part of the Geographic Survey Institute's (GSI) permanent GPS network in Kyushu, Japan. All permanent stations host dual-frequency GPS receivers, therefore all station coordinates are computed using the ionosphere-free combination. These results are considered as ground true for experimental comparisons.

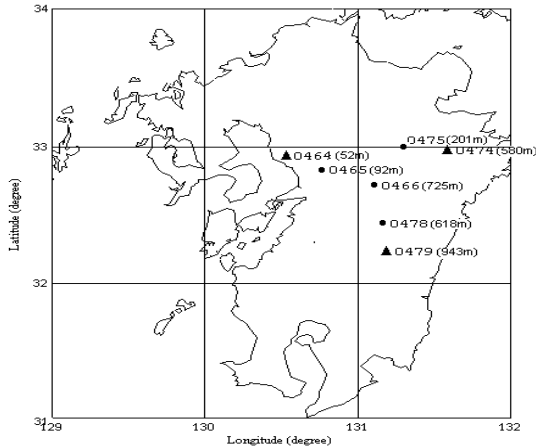
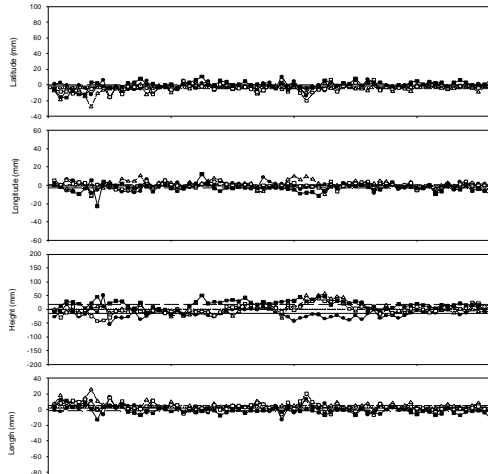


Figure 6 Some stations in GSI's Kyushu GEONET network, Japan.



4.1 Ambiguity resolution for newly risen satellites - the reference station

40 satellite rises (cutoff angle was 15°) were observed at stations 0479 and 0475 (90km baseline) on DoY 272, 1997. The characteristic that the double-differenced ionospheric and tropospheric delays are similar between successive days and on previous epochs has been used in the data processing strategy described earlier to aid ambiguity resolution. The result of this test is illustrated in Figure 7. The x-axis is the newly risen satellite numbers and the y-axis is the epoch numbers for correct ambiguity resolution (sample rate was 30sec). Applying the proposed polynomial model for ambiguity resolution, the estimation time can be lessened and the success rate can be improved, as indicated by the dark colour.

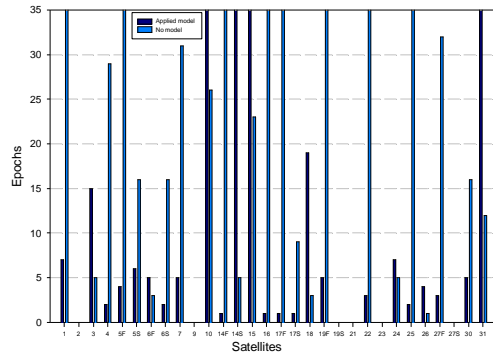
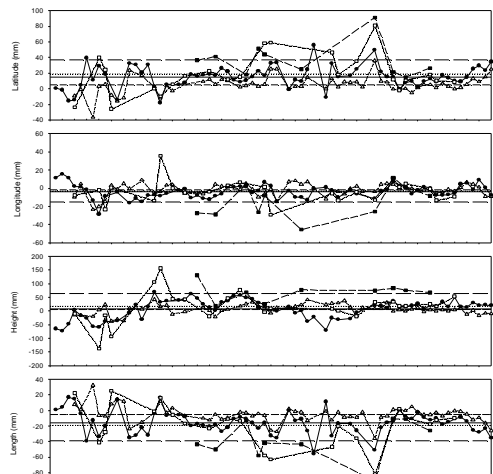


Figure 7. Number of epochs needed for ambiguity resolution for newly risen satellite (cutoff angle was 15°).

4.2 Positioning results - the user stations

To test the performance of the linear combination observation model for medium-range, GPS rapid static positioning using single-frequency user receivers. The results using four baselines 0478-0479 (23km), 0466-0478 (32km), 0466-0479 (55km) and 0465-0479 (77km) are compared. Figure 8 displays the comparison of results with network corrections, or without the incorporation of correction terms, for 40 minutes observation sessions. The higher accuracy results are obtained when the correction terms are applied in the baseline processing. The left hand graph shows the result with correction terms. It can be seen that the mean offsets and standard deviation of the variations in the latitude, longitude and baseline length components are less than 5mm, and 1-2cm for the height



component. The right hand graph displays the results obtained without applying the correction terms. The mean offsets and the standard deviations of the variations in the latitude, longitude and baseline length component are about 3-4cm, and 6-7cm for the height component. It can
(Results using proposed procedure)

be concluded that the accuracy of the baseline vectors is significantly improved when using the proposed network-based positioning strategy (Table 1 & 2).

(Conventional Method)

Figure 8 white triangles (23km baseline, mean: short dash line); solid circles (32km, mean: solid line); white squares (55km, mean: dotted line); solid squares (77km, mean: long dash line).

Table 1. The mean offsets (in mm) and standard deviation (in mm) for each session's (40min) results over two days, for the four baselines in the Kyushu GSI network, using the multiple reference station methodology.

Baseline Components	0478-0479 (23km)		0466-0478 (32km)		0466-0479 (55km)		0465-0479 (77km)	
	Mean	StD	Mean	StD	Mean	StD	Mean	StD
Latitude	-4.8	4.8	-1.4	4.7	3.4	5.0	0.2	4.8
Longitude	0.7	4.5	-0.5	3.7	-0.2	3.4	-2.9	4.5
Height	-1.9	20.0	-14.6	16.6	1.3	16.9	17.6	15.0
Length	4.8	4.7	1.4	4.6	3.3	4.9	-1.4	4.4

Table 2. The mean offsets (in mm) and standard deviation (in mm) of variations for each session's (40min) results without corrections from the multiple reference station methodology.

Baseline Components	0478-0479 (23km)		0466-0478 (32km)		0466-0479 (55km)		0465-0479 (77km)	
	Mean	StD	Mean	StD	Mean	StD	Mean	StD
Latitude	5.0	10.9	15.3	14.5	18.9	25.70	37.1	22.5
Longitude	-1.7	6.7	-3.7	7.6	-3.5	12.7	-15.4	18.0
Height	6.0	17.8	8.2	34.1	17.3	59.0	64.1	35.4
Length	-5.1	10.6	-15.6	14.8	-19.0	26.2	-38.7	24.9

5. Concluding remarks

A linear combination functional model for network-based GPS positioning has been proposed in which the orbit biases and the ionospheric delay terms are eliminated and, in addition, the tropospheric delay, multipath and observation noises can be reduced. A real-time ambiguity resolution technique for newly risen satellites is proposed here, in which the ionospheric and tropospheric delays are estimated using information from currently tracked satellites and previous days' tracking. The carrier phase corrections can then be determined

using multiple GPS reference stations and the broadcast ephemeris. The results show that the generated corrections do significantly improve the positioning results.

Acknowledgments. The third author is supported by a study grant from Taiwan's Institute of Earth Sciences, Academia Sinica, to undertake research towards a Ph.D. at The University of New South Wales.

References

- Han, S., Carrier phase-based long-range GPS kinematic positioning, *PhD Dissertation*, UNISURV S-49, School of Geomatic Engineering, The University of New South Wales, Sydney, Australia, 185pp, 1997.
- Han, S. and C. Rizos, GPS network design and error mitigation for real-time continuous array monitoring systems, *Proc. 9th Int. Tech. Meeting of the Satellite Division of the U.S. Inst. of Navigation GPS ION '96*, Kansas City, Missouri, 17-20 September, 1827-1836, 1996.
- Han, S. and C. Rizos, An instantaneous ambiguity resolution technique for medium-range GPS kinematic positioning, *Proc. 10th Int. Tech. Meeting of the Satellite Division of the U.S. Inst. of Navigation GPS ION '97*, Kansas City, Missouri, 16-19 September, 1789-1800, 1997.
- Li, Z. and Y. Gao, Improving ambiguity resolution for a regional area DGPS system using multiple days of data, *Proc. 11th Int. Tech. Meeting of the Satellite Division of the U.S. Inst. of Navigation GPS ION '98*, Nashville, Tennessee, 15-18 September, 399-406, 1998.
- Odiijk, D., Stochastic modelling of the ionosphere for fast GPS ambiguity resolution, *Abstracts of IUGG'99*, Birmingham, U.K., 19-24 July, 406, 1999.
- Schaer, S., G. Beutler, and M. Rothacher, The impact of the atmosphere and other systematic errors on permanent GPS networks, *Abstracts of IUGG'99*, Birmingham, U.K., 19-24 July, 406, 1999.