GPS Analysis with the Aid of Wavelets

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BIOGRAPHY

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ABSTRACT

The classical least-squares processing of GPS measurements generates residuals, which contains the signature of both unmodelled systematic biases and random measurement noise. It is desirable to extract (or minimise) the systematic biases contained within the GPS measurements. This would be relatively straightforward if there were some apriori knowledge of the phenomena related to these errors. Common ways of dealing with this problem include (i) changes to the stochastic modelling, and (ii) redefinition of the functional model.

In this study, we apply a method based on *wavelets* to decompose GPS double-differenced residuals into a low-frequency bias term and a high-frequency noise term. The extracted bias component is then applied directly to the GPS measurements to correct for this term. The remaining terms, largely characterised by the GPS range measurements and high-frequency measurement noise, are expected to give the best linear unbiased solutions from a least-squares process. A robust VCV estimation, using the MINQUE procedure, controls the formulation of the stochastic model. The results show that this method can improve both the ambiguity resolution and the accuracy of the estimated baseline components.

1. INTRODUCTION

The least-squares estimation method is usually employed for the processing of GPS measurements. The least-squares method is based on the formulation of a mathematical model consisting of the functional model and the stochastic model. If the function model is adequate, the residuals obtained from the least-squares solution should be randomly distributed. However, the GPS measurements are contaminated by several kinds of errors or biases such as the orbital error, the atmospheric biases, multipath disturbance and receiver noise. This would be relatively straightforward if there were some apriori knowledge of the phenomena related to these errors. As this is not the case, the least-squares method generates residuals which contain the signature of both unmodelled systematic biases and random measurement noise. It is desirable to extract (or minimise) the systematic biases contained within the GPS measurements. Recently, some wavelet-based techniques have been introduced in the field of GPS data processing (e.g. Collin & Warnant, 1995; Fu & Rizos, 1997; Ogaja et al., 2001; Satirapod, 2001). The methods introduced have, for example, addressed some of the potential applications such as signal denoising, outlier detection, bias separation and data compression.

This paper proposes a new method based on a *wavelet* decomposition technique and a robust estimation of the VCV matrix. The wavelet technique is first applied to decompose the GPS double-differenced residuals into the low-frequency bias and high-frequency noise terms. The extracted bias component is then applied directly to the GPS measurements to correct for the trend introduced by this error component. The remaining terms, largely characterised by the GPS range measurements and high-frequency measurement noise, are expected to give the best linear unbiased solutions from a least-squares process. The simplified MINQUE procedure (Satirapod et al, 2001) is applied to formulate the stochastic model.

The paper is organised as follows. In section two, the theory of wavelet decomposition and its application to GPS data processing is outlined. A discussion of the experimental results and analysis are presented in section three. Concluding remarks are given in section four.

2. WAVELETS

2.1 Theory

Wavelet theory provides a unified framework for a number of techniques, which have been developed independently for various signal processing applications. It has potential applications in filtering, subband coding, data compression and multi-resolution signal processing. In particular, the Wavelet Transform (WT) is of interest for the analysis of *non-stationary* signals such as GPS measurements, because it provides an alternative to the classical Fourier Transform (FT) which assumes stationarity in signals. It can be viewed as an extension to Fourier analysis that are well-suited for characterising signals whose spectral character changes with time. Such signals are not well represented in time and frequency by the Fourier Transform methods. The method of wavelet analysis is closely related to the time-frequency analysis based on the Wigner-Ville distribution (Olivier & Vetterli, 1991).

The WT involves representing general functions in terms of simple, fixed building 'blocks' at different scales and positions (Wickerhauser, 1994; Daubechies, 1990). These 'blocks' are actually a family of wavelet functions (or wavelet basis) generated from a prototype function, called a "mother" wavelet, by translation and scaling operations. That is, the signal is mapped to a time-scale plane that is analogous to the time-frequency plane used in the Fourier Transform.

Multi-resolution analysis provides a formal approach to constructing the wavelet basis. The idea of multi-resolution analysis is to write a function as a limit of successive approximations, each of which is a smoother version of the function. The subspaces contained within each other are meant to convey the notion of fine to coarse resolution, with the smoothness achieved through removal of some level of detail. For example, if the sub-space $V_{-1d}V_{0d}V_{1d}$... and W_0 is the orthogonal compliment of $V_{0d}V_{-1}$,

$$W_j \otimes V_j = V_{j+1}$$

The W_j contains the detailed information as the resolution goes from a finer (larger j) to a coarser (lower j) one. The subspaces, V_j, each contain the best approximation at a particular resolution, that is,

$$\lim_{j \to \infty} V_j = \bigcup_{j \to -\infty}^{\infty} V_j$$

and there will be information loss as the resolution gets coarser (j = ..., -3, -2, -1, 0), that is, in the limit of lowest resolution, the signal is approximated by 0:

$$\lim_{j \to \infty} V_j = \bigcup_{j \to -\infty}^{\infty} V_j = \{0\}$$

There are several types of Wavelet Transforms: for continuous signals, the time and scale parameters are continuous, leading to a Continuous Wavelet Transform (CWT). If the time and scale parameters are chosen to be discrete, this will give rise to a wavelet series expansion and and hence a Discrete Wavelet Transform (DWT) for a discrete signal. As the scale parameter grows, the signal dilates more, and like a map, the image or Wavelet Transform gives a more 'global' or low-frequency view. The translation parameter serves to shift the function along the time axis. A special case is developed by discretisation of the time-scale parameters. That is, if $a = 2^{-j}$ and $b = k2^{-j}$, the corresponding wavelets become a function of two integer parameters, *j* and *k*. For this case, the wavelets form a *dyadic series*.

Almost any function can be a prototype function, as long as it satisfies certain admissibility conditions. Daubechies (1990), for example, introduced a set of orthonormal wavelets, and more recently, a new family of non-orthogonal wavelets have been introduced by other authors. In general, the selection of the wavelet that best decomposes the data remains a research topic of its own.

2.2 Application of Wavelets to GPS Data Processing

A previous study by Fu & Rizos (1997) has outlined some of the applications of wavelets to GPS data processing. According to this study, the GPS bias terms such as multipath and ionospheric delay behave like low-frequency noise and the measurement noise as high-frequency noise. Hence, the GPS bias terms are concentrated in the narrow low-frequency band and a high frequency resolution is needed to identify them. The Wavelet Transform can be used to achieve enough frequency resolution to discriminate these terms in the original GPS measurement. Figure 1 illustrates that process.

The key is to find or design the best *mother wavelet* to use in the transform. The mother wavelet is scaled in time (dilated or compressed) and also shifted in time to effectively scan across the time-domain signal. Compressing the mother wavelet's time duration (width) effectively creates a high-pass filter (HPF) for extracting the high-frequency components of the analysed signal whereas dilating it creates a low-pass filter (LPF) for extracting the low-frequency components of the analysed signal.

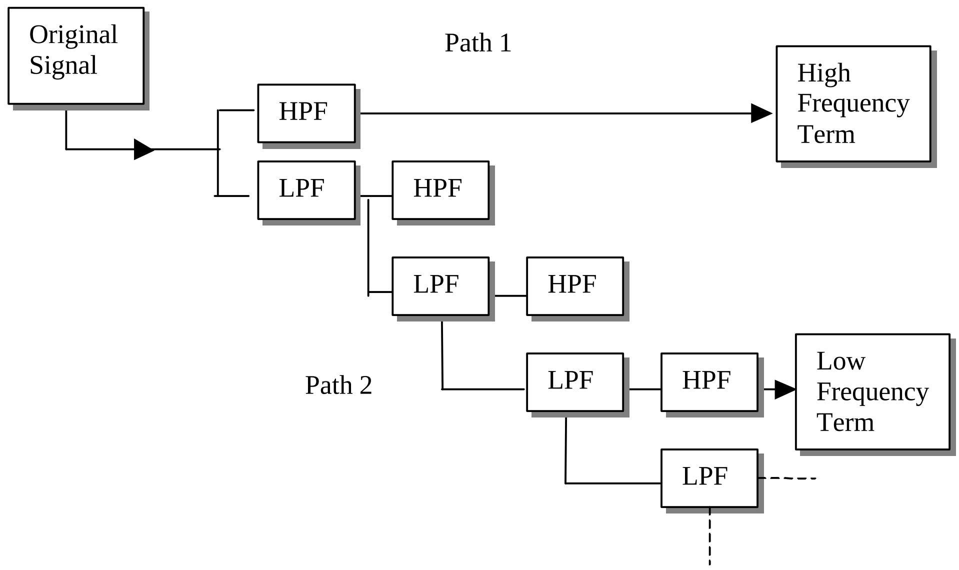

Figure 1. Applying a narrow daughter wavelet to the original signal is equivalent to applying a high-pass filter, which completes path 1. Extracting the leading low frequency requires applying a number of daughter wavelets that are wider than the signal you need to match, then applying a final daughter wavelet that becomes a high-pass filter, completing path 2.

A mother wavelet that approximates the bias term such as *multipath* is selected and dilated before performing the transform. To get the wavelet coefficients of sufficient magnitude to extract the bias term, the Wavelet Transform software has to process the wavelet a number of times, e.g. n times. For the first n-1 times the transform effectively passes the signal through a low-pass filter. On the n^{th} time the transform would produce coefficients substantial enough to extract the remaining low-frequency signal through the high-pass filter. At that point the width of the wavelet becomes long enough for its frequency to be below that of the bias term, and thus the final stage is a high-pass filter. The combination of the n-1 low-pass filters and the final high-pass filter creates a *bandpass filter*.

Figure 2 is an example of results after applying the process in Figure 1 to DD float ambiguity carrier phase residuals for a given pair of satellites. Path-1 indicates that the Wavelet Transform required just one high-pass filter to extract the high-frequency component of the residuals. Path-01, however, corresponds to two filter banks used to extract the corresponding high-frequency term at a different resolution level. The zero indicates that the signal passed through one low-pass filter before the transform could apply a high-pass filter. Similarly, Path-00 corresponds to two filter banks of low-pass filters only for extracting the low-frequency component.

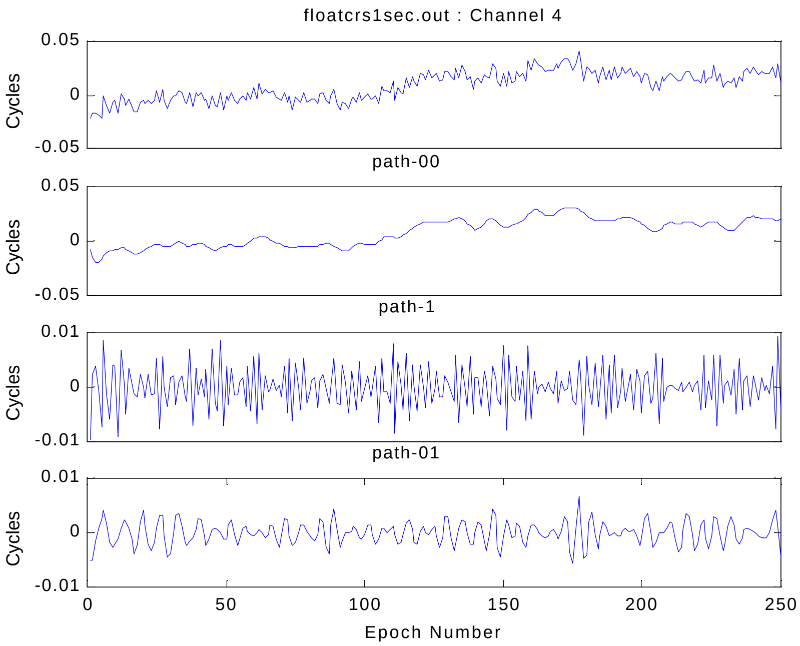

Figure 2: **First row:** DD float ambiguity carrier phase residuals (original signal); **second row:** low-frequency component; **third row:** high-frequency component; **fourth row:** high-frequency component (at higher resolution).

Once the wavelet application for extracting the low-frequency term such as *multipath* has been developed, it can be programmed to continuously process the GPS data affected by such biases.

3. EXPERIMENTAL RESULTS

In this section, results processed from real GPS data are presented, to demonstrate the usefulness of the proposed method of wavelet decomposition in extracting the low-frequency bias term. Static GPS data was analysed and the results from the processing are discussed.

3.1 Data Acquisition

The data set used here was collected on 7 June 1999 using two Ashtech Z-XII receivers at a sampling interval of 1 second. The receivers were mounted on pillars that are part of a first-order terrestrial survey network. The known baseline length between the two pillars is 215.929 ± 0.001m. This will be used as the ground truth to verify the accuracy of the results. A 30-minute span of data was cut from the original data set and resampled every 15 seconds. Six satellites (PRNs 2, 7, 10, 13, 19, and 27) were selected, as they were visible during the entire selected observation period. All data were first processed using the standard GPS data processing method to check the data quality. In the data processing step, satellite PRN2 was selected as reference satellite to form the double-differenced observables since it had the highest elevation. Double-differenced (DD) residuals for various satellite pairs are shown in Figure 3. The DD residuals indicate the presence of some significant multipath errors for satellite pairs PRN 2-7 and 2-19.

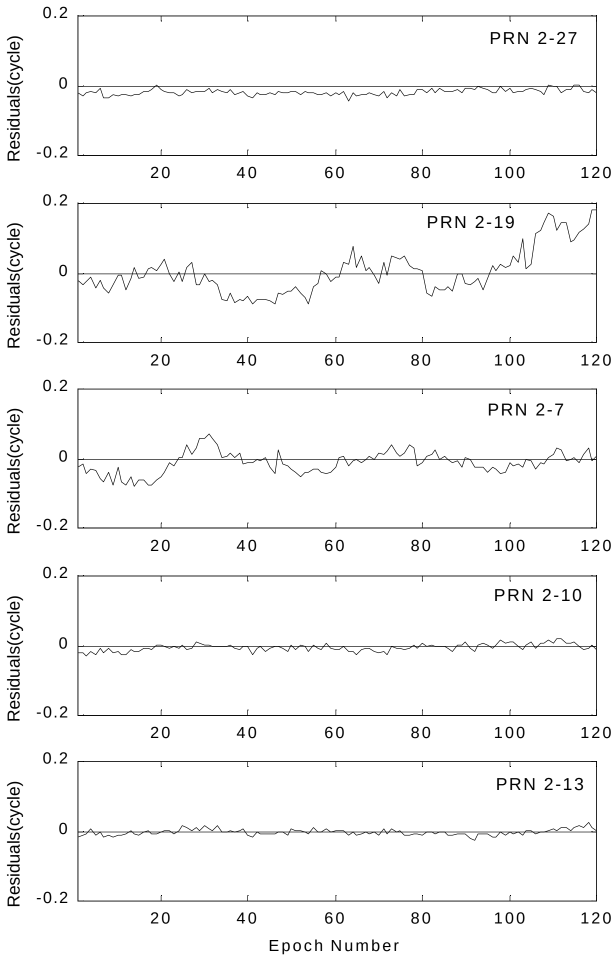

Figure 3: DD residuals obtained for the Ashtech receiver.

3.2 Data Processing Step

For convenience, the data set was divided into three batches, each of ten minutes length. Each batch was first processed using a standard GPS data processing method and treated as an individual session. The wavelet technique was then used to decompose GPS double-differenced (ambiguity-free) residuals into the low-frequency bias and the high-frequency noise terms for each batch (see Figure 4, for example). The extracted bias component was applied directly to the GPS measurements to correct for this term, and the simplified MINQUE procedure was then employed to estimate the variance-covariance matrix of the measurements. The results obtained from the standard method and the proposed method are discussed in the next section.

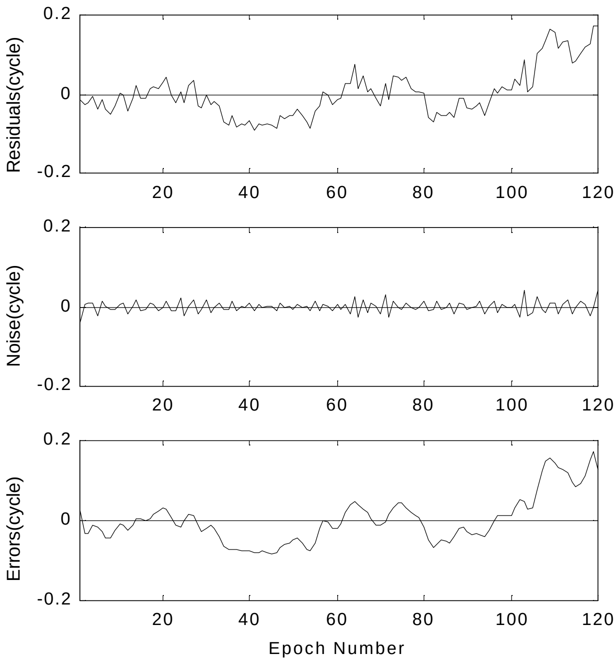

Figure 4: Signal extraction using wavelets. Top: Original DD residuals.
Middle: Extracted noise component. Bottom: Extracted systematic component.

3.3 Analysis of Results

Results obtained from the processing described in the previous section have been analysed from two points of view: ambiguity resolution and the estimation of baseline components. For reliable ambiguity resolution, the difference between the best and second-best ambiguity combination is crucial for the ambiguity discrimination step. The F-ratio is commonly used as the ambiguity discrimination statistic and the larger the F-ratio value, the more reliable is assumed the ambiguity resolution. The critical value of the F-ratio is normally (arbitrarily) chosen to be 2.0 (e.g. Euler & Landau, 1992). The ambiguity validation test can also be based on the newly-developed statistic W-ratio (Wang et al., 1998). In a similar fashion, the larger the W-ratio value, the more reliable the ambiguity resolution is assumed. The values of these statistics obtained from the data processing step are shown in Figure 5. The top plot indicates the F-ratio statistic, while the bottom plot represents the W-ratio statistic, where each group of columns represents the solution obtained from the individual sessions. As can be clearly seen, the F-ratio and W-ratio values obtained from the proposed method are larger compared to those from the standard method. This indicates that the reliability of the resolved ambiguities has been significantly improved.

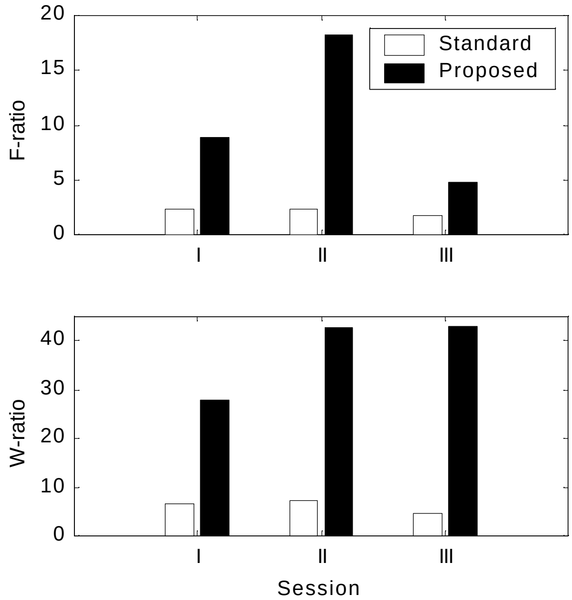

Figure 5: F-ratio and W-ratio statistics in ambiguity validation tests
Top: F-ratio value. Bottom: W-ratio value.

In the case of the estimated baseline components, the results are presented in Table 1. The results show that the proposed method produces more reliable estimated baseline components. This is confirmed by comparing the estimated baseline lengths obtained from both methods to the known baseline length. The values of the estimated baseline length obtained from the proposed method are much closer to the ground truth value than those obtained from the standard method. In addition, the maximum difference in the height component between sessions is up to 19.3 mm when the standard GPS data processing method is used. This is reduced to 9.3 mm when the proposed method is used.

Table 1: Estimated baseline components

Session	Methods	Estimated baseline components (m)			Baseline lengths (m)
		North	East	Height	
I	Standard	-188.5131	105.2933	0.5107	215.9262
	Proposed	-188.5147	105.2933	0.5121	215.9276
II	Standard	-188.5135	105.2932	0.5075	215.9265
	Proposed	-188.5154	105.2925	0.5099	215.9278
III	Standard	-188.5068	105.2954	0.4914	215.9217
	Proposed	-188.5132	105.2961	0.5028	215.9276

In a further investigation, the DD (ambiguity-fixed) residuals were decomposed into their respective high and low-frequency terms. The extracted systematic component was applied to the GPS measurements in the same way as in the above method. The results showed an improvement in statistics in ambiguity validation tests. However, the estimated baseline components obtained from this method exactly matched those obtained from the standard method.

4. CONCLUDING REMARKS

A new method of GPS data processing based on the wavelet decomposition and the robust estimation of VCV matrix has been developed. The initial results from the proposed method indicate that both the ambiguity resolution and the accuracy of estimated baseline components are improved. Further tests will be carried out to investigate the degree of effectiveness of this method.

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