GPS and GLONASS Integration: Modelling and Ambiguity Resolution Issues

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ABSTRACT

The integration of GPS with GLONASS may be considered a major milestone in satellite-based positioning, because it can dramatically improve the reliability and productivity of said positioning. However, unlike GPS, GLONASS satellites transmit signals at different frequencies, which results in significant complexity in terms of modelling and ambiguity resolution for integrated GPS and GLONASS positioning systems. In this paper, a variety of mathematical and stochastic modelling methodologies and ambiguity resolution strategies are analysed, and some remaining research challenges are identified. The exercise, of developing mathematical models and processing methodologies for integrated systems based on more than one satellite system, is a valuable one as it identifies crucial issues concerned with the combination of ANY two or more microwave positioning systems, be they satellite-based or terrestrial. Hence these are experiences that can be applied to future projects that might integrate GPS with Galileo, or GLONASS and Galileo, or all three.

INTRODUCTION

The Global Positioning System (GPS) is playing an ever increasing role in both surveying and navigation. It is well known that, for such satellite-based positioning systems, the accuracy, availability and reliability of the positioning results are very dependent on the number of satellites being tracked. However, in some situations, such as in urban canyons and in deep open-cut mines, the number of visible satellites may not be sufficient to perform the positioning function. One possible strategy for increasing the *availability* of satellites is to integrate GPS with another functioning satellite-based positioning system, such as the GLObal NAvigation Satellite Systems (GLONASS) operated by the Russian Federation. Like GPS, GLONASS has a great potential for precise navigation and geodetic applications (Zarraoa, *et al.*, 1998). This potential was demonstrated, in part, during the International

GLONASS Experiment-IGEX98 (Slater *et al.*, 1998; Willis *et al.*, 1999), and is being realised during the ongoing international GLONASS Service - Pilot Project (Reigber, 2000).

In both the GPS and GLONASS systems, two fundamental measurements can be decoded from the satellite signals, namely the *pseudo-ranges* and the *carrier phases*. As is well known, carrier phase measurements are much more precise than pseudo-ranges, thus they are the primary measurements for precise positioning. However, the carrier phase measurements are "ambiguous", with the "ambiguity" (the integer number of signal wavelengths between satellite and the receiving antenna) being an unknown value *a priori*. In GPS stand-alone positioning, the integer carrier phase ambiguities can be resolved in the relative mode using double-differencing procedures (Bossler *et al.*, 1980), also called 'differential interferometry' in radio astronomy (Counselman *et al.*, 1972). This principle can also applied to integrated GPS and GLONASS positioning.

In combined GPS and GLONASS data processing, in a manner similar to GPS stand-alone positioning, unknown parameters, *including baseline components and ambiguities*, are commonly estimated using least-squares or Kalman filtering techniques, both of which are based on so-called Gauss-Markov models comprising a mathematical (functional) model and a stochastic model. The *reliability* of the estimated results, therefore, is dependent on the correct definition of both the mathematical and stochastic models.

However, in combined GPS and GLONASS data processing, the tasks of defining the mathematical and stochastic models are difficult to accomplish. For example, due to the multiple frequencies of GLONASS carrier phases, standard double-differencing (DD) procedures cannot cancel receiver clock errors. The resulting design matrix has a rank deficiency, and thus the normal matrix becomes singular. Consequently, in theory, the ambiguity parameters cannot be separated from the receiver clock errors (Wang, 1999a). Therefore, some specific mathematical modelling strategies must be employed to remove the singularity within the normal matrix. On the other hand, the common practice of assuming that raw GPS and GLONASS measurements are statistically independent and have the same accuracy is certainly not realistic. The raw measurement errors are always spatially correlated (that is why the measurement differencing techniques are effective). Moreover, the measurements obtained from different satellites (even from different systems) have varying degrees of accuracy. Therefore, in a combined GPS and GLONASS positioning system the measurements will always have a heteroscedastic and correlated error structure, which needs to be accounted for in the stochastic model(s).

During the last decade much effort has been invested in order to develop reliable positioning and navigation systems using signals from the combined GPS and GLONASS satellite constellation. A variety of mathematical and stochastic modelling methodologies and ambiguity resolution procedures have been proposed in literature. This paper contributes to the review and analysis of some of these modelling and ambiguity resolution options.

MATHEMATICAL MODELLING

The *Mathematical models* describe relationships between the unknown parameters and measurements. At the beginning of the mathematical modelling procedure, one usually will take into account as many unknown parameters as possible to make sure that the mathematical model to be established reflects reality. For short baselines, the SD GPS/GLONASS code and carrier phase measurements can be modelled as follows:

$$R_{km}^{p} = \rho_{km}^{p} - cdt_{km} + \delta_{km}^{p} + \alpha_{km}^{p} + \tau_{km}^{p} \qquad (1)$$

$$\varphi_{km}^{p} = \frac{1}{\lambda_p}\rho_{km}^{p} - \frac{c}{\lambda_p}dt_{km} + N_{km}^{p} + \gamma_{km}^{p} + \omega_{km}^{p} + \varepsilon_{km}^{p} \qquad (2)$$

where the subscripts k and m identify the receivers, and the superscript p denotes the satellite; r is the topocentric range to the satellite; l is the wavelength of the carrier phase frequency; dt is the receiver clock error; c is the speed of light; N is the integer ambiguity parameter; δ and γ are the inter-channel hardware biases; α is the inter-system (GPS/GLONASS) bias; ω is the initial phase; and ε is the unmodelled bias or error term. Compared with the model discussed in Kozlov & Tkachenko (1998), the inter-system bias term α has been added to equation (1).

It should be noted, however, that in practice, due to the lack of enough geometric information contained within the measurements, not all the unknown parameters in equations (1) and (2) can be feasibly estimated. To overcome this difficulty, three basic modelling strategies are used, which are discussed below.

The first strategy is that a specific modelling procedure may be developed to reparameterize or to eliminate some unknown parameters in the mathematical model. For example, as with the integer ambiguity parameter, the initial phase of a receiver heterodyne is constant (it could be different for the two channels if two different heterodynes are used for the GPS and GLONASS channels). Therefore, in forming the DD (GPS-GPS, GLONASS-GLONASS) ambiguity parameters for the SD carrier phase model, the initial phase term for GPS (or GLONASS) can be lumped with the SD GPS (or GLONASS) ambiguity parameter for the GPS (or GLONASS) reference satellite. Actually, if the double-differencing procedure is used, this SD ambiguity parameter, together with the initial phase term, could be eliminated from the DD carrier phase model:

$$\varphi_{km}^{pq} = \frac{1}{\lambda_p}\rho_{km}^{p} - \frac{1}{\lambda_q}\rho_{km}^{q} - \left(\frac{c}{\lambda_p} - \frac{c}{\lambda_q}\right)dt_{km} + N_{km}^{pq} + \gamma_{km}^{pq} + \varepsilon_{km}^{pq} \qquad (3)$$

In equation (3), the inter-channel biases for the GPS carrier phases may also be eliminated because these biases are considered to be identical for all the GPS channels.

The second strategy is to use some ‘extra information’ within the positioning system in order to calibrate the measurements. For example, due to the non-constant group delay characteristic of the GLONASS RF front-end, the inter-channel hardware biases vary across the different GLONASS channels (Felhauer, 1997). These biases were identified at the beginning of GLONASS receiver development (Raby & Daly, 1993), and have been also noticed in both GLONASS pseudo-ranges and carrier phases from various receivers (for example, Dodson *et al.*, 1999; Jonkman *et al.*, 1998; Kozlov & Tkachenko, 1998; Pratt *et al.*, 1997; Tujii *et al.*, 1999; Wang, 1998a). Some GLONASS receiver manufacturers are investigating strategies to reduce the magnitude of inter-channel biases (for example, Fellhauer, 1997; Neumann *et al.*, 1999). It is also possible to estimate the biases using the GPS carrier phase measurements *when* both the DD GPS and DD GLONASS ambiguities are fixed to their correct integer values (Kozlov & Tkachenko, 1998; Rapoport *et al.*, 1999; Walsh & Daly, 1996; 1998). In Povalyaev (1997), the GLONASS inter-channel biases are separated into a constant term and a frequency-dependent term and, thus, can be calibrated using a linear function, resulting in the following model:

$$\varphi_{km}^{pq} = \frac{1}{\lambda_p}\rho_{km}^{p} - \frac{1}{\lambda_q}\rho_{km}^{q} - (\frac{c}{\lambda_p} - \frac{c}{\lambda_q})d\bar{t}_{km} + N_{km}^{pq} + \varepsilon_{km}^{pq} \qquad (4)$$

Compared with equation (3), equation (4) actually lumps the receiver clock error and the frequency-dependent part of the inter-channel biases together as a new parameter $d\bar{t}_{km}$, whilst the constant inter-channel bias term is eliminated in the double-differencing procedure.

The third strategy is to randomize the unmodelled error terms. Some unknown terms in the model, compared to the measurement noise level, may not be significant, and thus could be neglected in the mathematical model. For example, modern GPS/GLONASS receivers may have very small phase inter-channel biases (Gourevitch *et al*. 1996). In many investigations, such as Kozlov & Tkachenko (1998), Leick *et al*. (1998), Rapoport (1997) and Wang (1998a), no GLONASS inter-channel biases and inter-system bias (δ, γ and α) are calibrated. Instead, such unmodelled terms should be taken into account in the stochastic model.

However, even without inter-channel biases and inter-system biases in the carrier phase model, as discussed earlier, the mathematical modelling for GLONASS measurements is still a challenging problem. The reason for this is that the between-receiver clock parameters appear in the DD carrier observations (see equation (3)). Therefore, modelling methodologies have to properly deal with the receiver clock errors. In principle, the proposed modelling methodologies can be considered to belong to one of two categories: a) those that *eliminate* the receiver clock errors; or b) those that require the *estimation* of the receiver clock errors. Furthermore, the proposed models include such unknown parameters as baseline components, ambiguities, and receiver clock error terms. However, in some situations unmodelled errors could be significant, as will be discussed later.

Eliminating Receiver Clock Errors

Three specific modelling methodologies have been proposed to remove the receiver clock error terms from equation (3) or (4). Previous discussion of these methodologies can be found in Leick (1998) and Wang (1998b).

The *first methodology* is to estimate the receiver clock errors from the single-difference (SD) pseudo-ranges, and then correct the DD carrier phase measurements accordingly (for example, Raby & Daly, 1993; Pratt *et al.*, 1997). This method actually divides the parameter estimation process into two stages. The estimated receiver clock parameters in the first stage may be contaminated by large pseudo-range errors, which may include the inter-channel hardware delays between GLONASS pseudo-ranges, the inter-system receiver dependent biases and multipath. This will inevitably increase the uncertainty of the estimated coordinates and ambiguities in the second stage, because the clock parameters are treated as having known (true) values. To reduce the effect of the inter-system bias, the receiver clock errors may be estimated using the SD GLONASS and DD GPS pseudo-ranges (Wang, 1998a; Han *et al.*, 1999).

The s*econd methodology* is to scale carrier phases into distances or a chosen frequency (for example, Leick *et al.*, 1995; Leick, 1998; Landau & Vollath, 1996). The Trimble receiver

4000SGL scales all GLONASS signals to GPS L1 signals (Povalyaev *et al.*, 1996). An advantage of using this strategy is that the receiver clock errors can be easily eliminated at the modelling stage.

If the chosen frequency is f_0 and its wavelength is l_0, the double-differenced (GPS-GPS, GLONASS-GLONASS) carrier phase observation equation is obtained as (for example, Leick, 1998; Wang, 1998b):

$$j^{pq}_{km,l_0} \equiv \frac{l_p}{l_0} j^{p}_{km} - \frac{l_q}{l_0} j^{q}_{km} = \frac{1}{l_0} r^{pq}_{km} + \overline{N}^{pq}_{km} + \overline{e}^{pq}_{km} \quad (5)$$

with

$$\overline{N}^{pq}_{km} = \frac{\lambda_q}{\lambda_0} N^{pq}_{km} + \frac{\lambda_p - \lambda_q}{\lambda_0} N^{p}_{km} \quad (6)$$

where ρ^{pq}_{km} is the double-differenced topocentric distances and N^{p}_{km} is the SD ambiguity.

In the derivation of equation (5), although both inter-channel biases and initial phases are neglected, the impact of neglecting these terms is not significant. According to equation (4), as with the receiver clock errors, the frequency-dependent part of the inter-channel biases are eliminated from equation (5). The constant part of the inter-channel biases can be lumped with the initial phase term (see, equation (9) in Povalyaev (1997)). Because the 'lumped' initial phase term between the two receivers is the same, there could be small errors which are caused by the GLONASS wavelength differences. The maximum value for the 'lumped' initial phase term is 0.5 cycles, whilst the maximum value for the differences between the GLONASS wavelengths is 1.5mm for the current L1 frequency plan or 0.85mm for the final L1 frequency plan. Therefore, these errors are negligible.

It can be seen from equation (5) that the choice of l_0 is arbitrary. Scaling carrier phases to that frequency with a wavelength of one metre (l_0=1m) is equivalent to scaling carrier phases to distances. Although this method does cancel out the clock errors completely from the DD equations, the resulting DD ambiguities ($\overline{N}^{pq}_{km}$) are, unfortunately, no longer integers. One possible remedy for this is to estimate one of the SD ambiguity terms using the pseudo-range data and then correct the associated carrier. However, the correct estimate of the SD ambiguities then relies on the availability of high quality pseudo-range data. Incorrect SD ambiguities may result in unreliable DD ambiguity resolution (Leick *et al.*, 1995; Leick, 1998). Furthermore, even if the DD ambiguities are fixed to their correct integer values, wrong SD ambiguity values calculated from the pseudo-ranges may still cause big errors in the final baseline solutions (Wang, 1999a; 1999d).

Two alternative methods, which are not sensitive to pseudo-range errors, to determine the SD ambiguity will be discussed later in the ambiguity resolution section.

The *third methodology* is intended to cancel the clock errors and, at the same time, to preserve the integer nature of the DD ambiguity parameters. This can be achieved when the following condition is satisfied:

$$\lambda_0 = \frac{\lambda_p}{k_p} = \frac{\lambda_q}{k_q} \tag{7}$$

where k_p and k_q are integers (Rossbach & Hein, 1996). In the derivation of condition (7), however, the inter-channel biases and initial phases are not taken into account. Also, the disadvantage of this method is that the magnitude of the resulting wavelengths is of the order of micrometres, and thus the associated integer ambiguities cannot be resolved. It could be argued that some suitable factors k_p and k_q can result in longer wavelengths. However, this remedy is not feasible. As shown in Wang *et al*. (1999), for the GLONASS-GLONASS satellite pair, the largest possible value of l_0 is 0.00126m for the current frequency allocation and 0.000593m for the final frequency plan. With such small wavelengths, it is still extremely difficult (if not impossible) to fix the associated ambiguities.

Estimating Receiver Clock Errors

It has been noted that the rank deficiency existing in the DD carrier phase measurement equations can be removed by adding SD pseudo-range measurements. This approach can produce an estimate of the (relative) receiver clock parameter, together with other parameters. Actually, if the SD pseudo-range measurements are included in the mathematical models, it is also possible to combine DD and SD carrier phase measurements in which the SD ambiguities are transformed into DD ambiguity parameters. This was first suggested by Walsh & Daly (1996), and explicitly discussed in Kozlov & Tkachenko (1998), Rapoport (1997), and Wang (1998a; 1999a).

In integrated GPS and GLONASS positioning, the double-differenced carrier phase measurements may be constructed in the forms: a) the GPS-GPS and GLONASS-GLONASS combination, called *the separated DD carrier phase formulation*, or b) the GPS-GLONASS DD carrier phase combination, called *the mixed DD carrier phase formulation.*

Generally, the mixed formulation is useful only when the initial phases for GPS and GLONASS channels are identical. This means that a common heterodyne is used for all the channels. It is also noted that the initial phase difference between the GPS and GLONASS channels could be estimated to a reasonable level of accuracy (Kozlov & Tkachenko,1998; Leick *et al*., 1998). The estimated difference could be used to calibrate the phase measurements after full loss of lock because the initial phase difference is thought to be constant until the receiver is turned off (Kozlov & Tkachenko, 1998). Therefore, this mixed formulation is also useful even when two different heterodynes are used for the GPS and GLONASS channels.

Similar to the DD carrier phase formulations, there are two possible options for forming the DD ambiguities in the SD carrier phase measurement equations: a) the *mixed DD ambiguity formulation*, i.e. selecting only one GPS or GLONASS satellite as the reference satellite; and b) the *separated DD ambiguity formulation*, i.e. only GPS-GPS and GLONASS-GLONASS DD ambiguities are constructed. Furthermore, the pseudo-range measurements from GPS and GLONASS can also be 'separated' in such a way to form the SD GPS and DD GLONASS or DD GPS and SD GLONASS pseudo-range measurements (the clock parameters are excluded from the DD pseudo-range measurement equations). These pseudo-range measurement formulations may have a different impact on ambiguity resolution and the final baseline solution. The following notation is employed to represent the different model formulations (Wang, 1998a):

$$[a][b]\_[c][d]\_[e] \qquad (8)$$

where

$[a]$ is the GPS pseudo-range mode;
$[b]$ is the GLONASS pseudo-range mode;
$[c]$ is the GPS carrier phase mode;
$[d]$ is the GLONASS carrier phase mode;
$[e]$ is the DD carrier phase or DD ambiguity formulation (reference) mode.

The measurement modes for $[a]$, $[b]$, $[c]$ and $[d]$ can be *S*, denoting the single-differencing mode or, *D*, denoting the double-differencing mode. $[e]$ can be *M*, for the mixed formulation approach or, *S*, in the case of the separated formulation approach. For example, the notation *DS_DD_S* represents the model which uses the double-differenced GPS pseudo-range, the single-differenced GLONASS pseudo-range, GPS-GPS and GLONASS-GLONASS double-differenced carrier phase measurements. With such a model formulation procedure, a total of 18 different models can be formed. These model formulations and their relative redundancy are listed in Table 1 (Wang, 1999b).

Table 1 Model formulations for integrated GPS-GLONASS positioning and their relative redundancy

Model Numbers	Model Names	Relative Redundancy* Static	Relative Redundancy* Kinematic
1	*SS_SS_S*	$3n_e-2$	$3n_e-2$
2	*SS_SD_S*	$2n_e$ - 1	$2n_e-1$
3	*SS_DS_S*	$2n_e$ -1	$2n_e-1$
4	*SS_DD_S*	N_e	N_e
5	*S D_SS_S*	$2n_e$ -2	$2n_e-2$
6	*SD_SD_S*	n_e -1	n_e-1
7	*SD_DS_S*	n_e -1	n_e-1
8	*SD_DD_S*	0	0
9	*DS_SS_S*	$2n_e$ -2	$2n_e-2$
10	*DS_SD_S*	n_e -1	n_e-1
11	*DS_DS_S*	n_e -1	n_e-1
12	*DS_DD_S*	0	0
13	*SS_SS_M*	$3n_e$ -2	$3n_e-2$
14	*SS_DD_M*	$2n_e-1$	$2n_e-1$
15	*SD_SS_M*	$2n_e$ -2	$2n_e-2$
16	*SD_DD_M*	n_e -1	n_e-1
17	*DS_SS_M*	$2n_e$ -2	$2n_e-2$
18	*DS_DD_M*	n_e -1	n_e-1

* n_e is the number of epochs in a solution.

As discussed above, the models containing M (13 to 18) are reliable only when the initial phases for the GPS and GLONASS channels are identical or have been otherwise calibrated.

Generally speaking, when neglecting the initial phases, inter-channel and inter-system biases, all the mathematical model formulations that include the SD GPS (or GLONASS) pseudo-range measurements are valid. It is noted that Model 1 and Model 12 have been implemented in commercial software packages (Kozlov & Tkachenko, 1998; Rapoport, 1997). However, since the error characteristics of GPS and GLONASS measurements are different, these possible mathematical models have different levels of performance in the case of ambiguity resolution and baseline determination. Based on model performance for ambiguity resolution, and an analysis of model sensitivity to the clock parameter, the optimal model has been identified as being Model 12 (Wang, 1998a).

This optimal model may also be justified by error analysis. Similar to the initial phases for carrier phases, the inter-system biases in pseudo-ranges have also been identified (Kozlov & Tkachenko, 1998). These are called receiver GPS-GLONASS biases in Dow *et al.* (1998), or receiver-specific biases in Habrich (1999). For the optimal model *DS_DD_S*, these inter-system biases and initial phases can be eliminated because the carrier phases and pseudo-ranges from the GPS and GLONASS systems are effectively 'separated'.

STOCHASTIC MODELLING

The *stochastic models* describe the statistical quality of the mathematical models, which are mainly defined by the covariance matrices of the mathematical model errors.

No model is 'perfect'. As in the case of GPS-only positioning systems, unmodelled errors or model errors of the SD and DD measurement equations in integrated GPS and GLONASS positioning systems also include satellite orbital errors, atmospheric delays, multipath effects, phase centre variations, inter-channel biases, etc. Orbital errors and atmospheric delays can, to a large extent, be eliminated for short baselines.

It is expected that differences between the GPS and GLONASS coordinate datums and time frames will result in model errors. Such problems could be addressed by a global network of monitoring stations, such as the International GLONASS Service (Reigber, 2000). For short baselines, the effects of differences in system datum and time scale are considered to be insignificant (for example, Laudau & Vollath, 1996; Pratt *et al.*, 1997).

Even using an optimal mathematical model there are still some unmodelled (or residual) systematic errors in the SD or DD differenced measurements, which are considered as randomised errors and must be accounted for in stochastic modelling. In combined GPS and GLONASS data processing, down-weighting the GLONASS measurements is a common practice (for example, Hall *et al.*, 1997; Kozlov & Tkachenko 1998; Tujii *et al.*, 1999; Wang, 1998a). Therefore, a realistic stochastic model is critical for integrated GPS and GLONASS positioning.

In static positioning, modern statistical methods, such as MINQUE (*Minimum Norm Quadratic Unbiased Estimation)*, have been successfully applied to estimate the covariance matrix for the double-differenced GPS measurements (Wang *et al.*, 1998b). Although the MINQUE method has some well defined properties, it requires an iterative procedure and the number of iterations depends on the properties of the data and model themselves. Thus, it is unsuitable for real-time kinematic positioning.

An online stochastic modelling method has recently been developed (Wang, 1998b; 1999c). This method is based on the Kalman filtering residuals:

$$v_i = z_i - H_i \hat{x}_i \tag{9}$$

where z_i is a vector of all the measurements used in epoch i; H_i is the design matrix relating to the measurements z_i; and $\hat{x}_i$ is the estimator for the unknown parameters generated by the Kalman filtering process at epoch i. By applying the error propagation law to equation (9), the covariance matrix of the residuals is derived:

$$Q_{v_i} = R_i - H_i Q_{\hat{x}_i} H_i^T \tag{10}$$

where R_i and $Q_{\hat{x}_i}$ are the covariance matrices for the measurements and the estimated unknown parameters respectively. The principle underlying this method is to make the elements of the actual covariance matrix of the filtering residuals consistent with their theoretical values. The actual covariances of the filtering residuals are approximated by their sample covariances. Therefore, the covariance matrix of the measurements is estimated as:

$$\hat{R}_i = \hat{Q}_{v_i} + H_i Q_{\hat{x}_i} H_i^T = \frac{1}{m}\sum_{k=0}^{m-1} v_{i-k} v_{i-k}^T + H_i Q_{\hat{x}_i} H_i^T \tag{11}$$

which can be used in the computation of epoch $i+1$. In equation (11), m is called *the width of moving windows*.

In practical applications, the filtering residuals should be computed from the ambiguity-fixed solutions because the float ambiguity parameters may absorb some unmodelled errors, which can be represented by the stochastic model. Testing indicates that the optimal width of the moving windows is in the range of 7-30 epochs with an interval of 1 second, and slightly depends on the redundancy of the positioning system (Wang, 1999c).

In many situations, the systematic errors existing in GPS and GLONASS measurements change slowly over time and thus the model errors are time-correlated. Simultaneously estimating both the space- and time-correlations in satellite measurements remains a challenging issue.

SPECIFIC AMBIGUITY RESOLUTUION STRATEGIES

General ambiguity resolution theory and algorithms have been discussed in, for example, Povalyaev (1976, 1978), Penzin (1990), Teunissen (1993, 1998), Hassibi & Boyd (1998), and Wang *et al*. (1998a). The developed theory and algorithms, however, are based on regular mathematical models, in which the design matrix is of full rank. In the case of GLONASS and integrated GPS/GLONASS positioning, a regular mathematical model will rely on the use of pseudo-ranges, although they are used in different ways. Unlike GPS, the effects of the inter-channel hardware delays on GLONASS pseudo-ranges are significant (Gourevitch *et al*., 1996; Pratt *et al*., 1997). Therefore, for high-precision geodetic applications of the GLONASS data, it will be also of great importance to develop suitable algorithms to process carrier phase data alone. Examples of work in this regard are reported in Habrich (1998), Rossbach & Hein (1996), Povalyaev (1997) and Wang (1999a, 1999d).

Based on the modelling strategy proposed by Leick *et al.* (1995), converting the GLONASS carrier phases to distances before forming the DD measurements can remove the receiver clock terms from the DD measurement equations. In such models, both SD and DD ambiguity parameters simultaneously appear in the DD measurement equations, resulting in a singularity in the normal matrix. Two different approaches have been proposed to tackle this problem: a) the two-level search approach (Wang, 1999a; 1999d); and b) the iterative search approach (Habrich, 1999; Habrich *et al.*, 1999).

Two-level search approach

When satellite p is chosen as the reference satellite for forming all the DD GLONASS carrier phases, N_{km}^{p} is a unique unknown SD ambiguity parameter in the data processing (see equation (6)). Because it is impossible to estimate both SD and DD ambiguities simultaneously, the SD ambiguity parameter is assigned an approximate value $\tilde{N}_{km}^{p}$, which may be determined from pseudo-range data processing (Leick, *et al.*, 1995; Landau, 1998). Then equation (5) is rewritten as ($\lambda_0 = 1$):

$$\overline{\varphi}_{km}^{pq} - (\lambda_p - \lambda_q)\tilde{N}_{km}^{p} = \rho_{km}^{pq} + \lambda_q N_{km}^{pq} + \varepsilon_{km}'^{pq} \tag{12}$$

With the SD ambiguity parameter fixed to a value, the DD ambiguities can be estimated and resolved using standard procedures. Because the incorrect SD ambiguity value may cause systematic errors in the DD carrier phase measurements, and thus result in unreliable DD ambiguity resolution, an optimal SD ambiguity value must be identified. The identification of the optimal (or most likely) SD ambiguity value is premised on the following two facts:

(1) the possible range or space of the SD ambiguity is sufficiently well known; and
(2) the closer the SD ambiguity parameter is to its correct value, the better the performance of the DD ambiguity resolution.

Therefore, by searching through all the possible integer SD ambiguity values (within a reasonable range), the best (most likely) SD ambiguity value can be identified, which is associated with the minimum value of the least-squares residuals. When the SD ambiguity value is fixed to its most likely value, the correct DD ambiguities can be more easily recovered. A mathematical relationship between any two DD ambiguity-float solutions based on different SD ambiguity values has been established, which makes the two-level search process more efficient (Wang, 1999d).

When the DD ambiguities are fixed to their correct integers, the SD ambiguity parameter in the final baseline solutions is fixed to its most likely value, or is treated as a real-valued parameter. However, there is no significant difference between the final solutions based on these two different treatments of the SD ambiguity parameter (Wang, 1999a; 1999d). In Han *et al.* (1999) and Habrich *et al.* (1999), the SD ambiguity parameter is also treated as a real-valued quantity.

This approach is mainly suitable for static, fast-static GPS/GLONASS data processing of short observation sessions, in which one satellite could be selected as the sole reference satellite to form all the double-differenced carrier phase observations. In theory, a multi-level search algorithm accommodating more reference satellites is also possible, although such an algorithm will be very complex.

Iterative search approach

In contrast to the two-level search approach, the iterative search approach is mainly designed for use in static GPS/GLONASS data processing with long observation sessions. Therefore, at the beginning of data processing, the parameter estimation is performed at the SD ambiguity level. There is a SD ambiguity parameter for each satellite. Thus equation (5) is rewritten as ($\lambda_0 = 1$):

$$\overline{\varphi}_{km}^{pq} = \rho_{km}^{pq} + \lambda_p N_{km}^{p} - \lambda_q N_{km}^{q} + \varepsilon'^{pq}_{km} \tag{13}$$

Similar to the DD carrier phase equations expressed in equation (5), equation (13) will also lead to a singular normal matrix. To remove this singularity, an *a priori* constraint (of, for example, 200 cycles) for each SD ambiguity parameter is introduced, which may be considered a *pseudo-measurement* of the SD ambiguity.

Carrier phase ambiguities are resolved at the double-difference level. In each iteration, all possible combinations of DD ambiguities are compared, and the best determined, which has the lowest formal error, is fixed to its ***correct*** integer. For instance, if the DD ambiguity N_{km}^{pq} is fixed to an integer N_0, then:

$$N_{km}^{pq} = N_{km}^{p} - N_{km}^{q} = N_0 \tag{14}$$

Thus, the following equation is obtained:

$$N_{km}^{p} = N_{km}^{q} + N_0 \tag{15}$$

Substituting equation (15) into equation (16) yields:

$$\overline{\varphi}_{km}^{pq} - \lambda_p N_0 = \rho_{km}^{pq} + (\lambda_p - \lambda_q) N_{km}^{q} + \varepsilon'^{pq}_{km} \tag{16}$$

In equation (16), the SD ambiguity parameter N_{km}^{p} is actually eliminated. After the elimination of the resolved ambiguity, more iterations are carried out until all possible DD ambiguities are fixed. The only one remaining SD GLONASS ambiguity parameter is treated as a real-valued unknown parameter in the final baseline solution.

Because the constraints, or pseudo-measurements, for the SD ambiguities are assumed to have low accuracy, a long observation session (≥ 1 hour) may be required to obtain reliable ambiguity resolution results (Habbrich *et al.*, 1999). On the other hand, if the explicit mathematical relationship between the iterated solutions can be derived, a more efficient implementation algorithm may be developed.

SUMMARY

The standard double-differenced procedure commonly used in GPS data processing cannot be applied to GLONASS-only, or combined GPS and GLONASS data processing, due to the multiple frequencies of the GLONASS ranging signals. This fact results in complexity in both the modelling and ambiguity resolution. Several mathematical and stochastic modelling methodologies, and specific ambiguity resolution procedures, have been analysed and remaining research issues have been identified.

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