Stochastic Assessment of GPS Carrier Phase Measurements for Precise Static Relative Positioning

Jinling Wang
Chalermchon Satirapod
Chris Rizos

School of Geomatic Engineering
University of New South Wales
Sydney, NSW 2052
Australia

Tel: 61 2 9385 4203
Fax: 61 2 9313 7493
email: Jinling.Wang@unsw.edu.au

Abstract

Global Positioning System (GPS) carrier phase measurements are used in all precise static relative positioning applications. The GPS carrier phase measurements are generally processed using the least-squares method, for which both functional and stochastic models need to be carefully defined. Whilst the functional model for precise GPS positioning is well documented in the literature, realistic stochastic modelling for the GPS carrier phase measurements is still both a controversial topic and a difficult task to accomplish in practice. The common practice of assuming that the raw GPS measurements are statistically independent in space and time, and have the same accuracy, is certainly not realistic. Any misspecification in the stochastic model will inevitably lead to unreliable positioning results. In this paper, a stochastic assessment procedure has been developed to take into account the heteroscedastic, space- and time-correlated error structure of the GPS measurements. Test results indicate that by applying the stochastic assessment procedure developed here, the reliability of the estimated positioning results is improved. In addition, the quality of ambiguity resolution can be more realistically evaluated.

Key Words: GPS, Carrier phase, Stochastic assessment, Relative positioning

1 Introduction

Since its introduction to civilian users in the early 1980's, the Global Positioning System (GPS) has been playing an increasingly important role in high-precision surveying and geodetic applications. As with traditional geodetic network adjustment, data processing for precise GPS static positioning is invariably performed using the least-squares method. To employ the least-squares method for GPS relative positioning, both the functional and stochastic models of the GPS measurements need to be defined. The *functional model*, also called the mathematical model, describes the mathematical relationships between the GPS measurements and the unknown parameters such as the ambiguity terms and the baseline components. The *stochastic model* describes the statistical properties of the measurements, which are mainly defined by an appropriate covariance matrix. Over the last two decades the functional models for GPS carrier phases have been investigated in considerable detail, and are well documented in the literature (for example, in such texts as Hofmann-Wellenhof et al., 1994; Leick, 1995; Rizos, 1997; Seeber, 1993; Teunissen and Kleusberg, 1998). However, accurate stochastic modelling for the GPS measurements is still both a controversial topic and a difficult task to implement in practice (Cross et al., 1994; Wang and Stewart, 1996).

In practice, the stochastic models of GPS measurements are mainly based on considerable simplifications. In current stochastic models it is usually assumed that all the one-way carrier phases or pseudoranges have the same variance, and that they are statistically independent. The time-invariant covariance matrix of the double-differenced (DD) measurements is then constructed using the error propagation law. In this covariance matrix the correlation coefficient between any two DD measurements is $+0.5$. This so-called 'mathematical correlation' is introduced by the double differencing process. To set up a simple stochastic model for DD measurements, it is further assumed that temporal correlations are absent. However, these assumptions are not realistic. As commented in, for example, Goad (1987), Gourevitch (1996), and Langley (1997), the GPS measurement errors are dominated by the systematic errors caused by the orbit, atmospheric and multipath effects, which are quite different for each satellite. Therefore the measurements obtained from different satellites cannot have the same accuracy due to varying noise levels. On the other hand, the raw measurements are spatially correlated due to similar observing conditions for these measurements (it is this fact that makes the double differencing procedure effective in mitigating measurement biases). Moreover, the time correlations may exist in the measurements because the residual systematic errors change slowly over time.

To model the *heteroscedasticity*, the accuracy of the one-way GPS measurements may be calculated using some approximate formula defined by the satellite elevation angle (for example, Euler and Goad, 1991; Gerdan, 1995; Han, 1997; Jin, 1996; Rizos et al., 1997), or signal-to-noise ratios (for example, Barnes et al., 1998; Brunner et al., 1999; Gianniou and Groten, 1996; Hartinger and Brunner, 1998; Langley, 1997; Talbot, 1988). Given the variances of the one-way measurements, the covariance matrix for the DD measurements is constructed using the error propagation law. However, in some cases, these two accuracy indicators, satellite elevation and signal-to-noise ratio, may not be reliable (Satirapod and Wang, 2000). A more rigorous statistical method, known as MINQUE (*Minimum Norm Quadratic Unbiased Estimation,* Rao, 1971), can be employed to estimate the stochastic model for the GPS DD measurements (Wang et al., 1998b). In this procedure temporal correlations are assumed to be absent in the GPS measurements.

The impact of *temporal correlations* on GPS baseline determination has been investigated in, for example, Vanicek et al. (1985), El-Rabbany (1994), Han and Rizos (1995) and Howind et al. (1999). In these studies all one-way measurements are considered to be independent, and having the same variance and same temporal correlation. It has been noted that the GPS measurement may have a heteroscedastic, space- and time-correlated error structure (Wang, 1998). Any misspecification in the stochastic models will result in unreliable positioning results.

In this paper, an iterative stochastic assessment procedure is proposed, in which all of the aforementioned error features of GPS measurements are taken into account. In the following sections the authors will first briefly describe the mathematical equations used in static GPS baseline data processing, and then discuss the estimation of variance-covariance components and the treatment of temporal correlations. Then details of the iterative stochastic modelling method will be presented. Applications of the proposed method will be demonstrated using a variety of GPS data sets.

2. Basic Equations for Processing GPS Carrier Phase Measurements

In precise GPS positioning, double-differenced (DD) carrier phase observables are usually formed because many systematic errors existing in the GPS measurements cancel, and the resultant DD observables have a simplified mathematical model. For short baselines, the DD carrier phases can be expressed as

$$\varphi_{uv}^{pq}(t) = \frac{1}{\lambda} Z_{uv}^{pq}(t) + N_{uv}^{pq} + e_{uv}^{pq}(t) \tag{1}$$

where the superscripts p and q denote satellites, and the subscripts u and v specify the receivers, and the indices t denote the epoch at which the data were collected. Z is the topocentric distance to the satellites, λ is the wavelength of the carrier wave, and N is the DD integer ambiguity. The term e represents all possible errors, including random noises of receivers and residual systematic errors such as unmodelled multipath effects, ionospheric and tropospheric delays, etc.

Assuming that the vector x contains all the unknown parameters necessary for baseline data processing, a set of linearized DD measurement equations for the ith satellite pair can be formed:

$$l_i = A_i x + e_i, \qquad i = 1, 2, ..., n \tag{2}$$

with

$$l_i = [l_i(1),\ l_i(2),\ . \ . \ ., \ l_i(s)]^T;$$

$$A_i = [A_i(1)\ \ A_i(2)\ \ . \ . \ ., \ A_i(s)]^T; \text{ and}$$

$$e_i = [e_i(1),\ e_i(2),\ . \ . \ ., \ e_i(s)]^T.$$

In equation (2), l_i is an $s \times 1$ vector of the observed minus computed DD carrier phase values; A_i is the design matrix corresponding to the measurements l_i; e_i is an $s \times 1$ vector of the error terms for the measurement l_i; n is the number of satellite pairs forming the DD observables; and s is the number of observation epochs.

By collecting all the linear(ized) DD carrier phase equations from the whole observation session, the functional (mathematical) model is then constructed:

$$l = Ax + e \tag{3}$$

with

$l = [l_1^T, l_2^T, ..., l_n^T]^T$;

$A = [A_1^T, A_2^T, ..., A_n^T]^T$; and

$e = [e_1^T, e_2^T, ..., e_n^T]^T$.

In practice, GPS measurements are usually assumed to have the same precision and to be statistically independent in time and space, i.e.

$$E[e_i(t)] = 0 \tag{4}$$

$$E[e_i(t) \cdot e_j(v)] = 0 \quad (t \neq v) \tag{5}$$

$$E[e_i(t) \cdot e_j(t)] = \sigma_{ij}^2 = \begin{cases} 4\sigma^2 (i \equiv j) \\ 2\sigma^2 (i \neq j) \end{cases} \tag{6}$$

where $i = 1, 2, ..., n$; and $t, v = 1, 2, ..., s$; and σ is the standard deviation of the one-way measurements. Then, a covariance matrix for all the DD observables l is constructed (for the case that all epochs have the same satellites):

$$C = \Sigma \otimes I_s = \sigma^2 \cdot Q \otimes I_s \tag{7}$$

with

$$\Sigma = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & . & . & . & \sigma_{1n}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & . & . & . & \sigma_{2n}^2 \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ \sigma_{n1}^2 & \sigma_{n2}^2 & . & . & . & \sigma_{nn}^2 \end{bmatrix} = \begin{bmatrix} 4\sigma^2 & 2\sigma^2 & . & . & . & 2\sigma^2 \\ 2\sigma^2 & 4\sigma^2 & . & . & . & 2\sigma^2 \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ 2\sigma^2 & 2\sigma^2 & . & . & . & 4\sigma^2 \end{bmatrix} = \sigma^2 Q,$$

where I_s is the $s \times s$ Identity matrix, and Q is a co-factor matrix. With the mathematical and stochastic models expressed by equations (3) and (7), the least-squares estimator of the unknowns, and the residuals of the measurements, can be obtained:

$$\hat{x} = [A^T(\Sigma^{-1} \otimes I_s)A]^{-1} A^T(\Sigma^{-1} \otimes I_s)l = [A^T(Q^{-1} \otimes I_s)A]^{-1} A^T(Q^{-1} \otimes I_s)l \quad (8)$$

$$\hat{e} = l - A\hat{x} \quad (9)$$

Clearly, the estimate $\hat{x}$ is independent of the variance factor. The optimal estimate of this variance factor is given by:

$$\hat{\sigma}^2 = \frac{\hat{e}^T(Q^{-1} \otimes I_s)\hat{e}}{f} \quad (10)$$

where f is the degree of freedom (Leick, 1995). The estimated variance factor $\hat{\sigma}^2$ is an indicator of the accuracy of the measurements in general. By substituting the unknown variance factor σ^2 with the estimated one $\hat{\sigma}^2$, the covariance matrix Σ of the measurements is replaced by the matrix $\hat{\Sigma}$. The covariance matrix for the estimate $\hat{x}$ is then written as:

$$C_{\hat{x}} = \hat{\sigma}^2[A^T(Q^{-1} \otimes I_s)A]^{-1} = [A^T(\hat{\Sigma}^{-1} \otimes I_s)A]^{-1} \quad (11)$$

It is easy to see that the estimator $\hat{x}$ and its covariance matrix $C_{\hat{x}}$ are dependent on the stochastic model adopted for the measurements. Any misspecifications of the stochastic model will lead to unreliable results, contrary to the optimal property of the least-squares solution. By using a misspecified stochastic model, the least-squares computations will produce unrealistic statistics, that could be used in ambiguity resolution and in the final baseline determination. Therefore, a realistic stochastic model for GPS baseline processing is critical and is discussed in detail in the next section.

3 Stochastic Assessment of Carrier Phase Measurements

From the data processing point of view, the stochastic model is essentially a fully distributed covariance matrix for all the measurements used in the least-squares estimation procedure. Generally, the magnitude of the elements of such a covariance matrix are unknown. Consequently, similar to the situation of the functional model, there are also unknown parameters in the stochastic model. A rigorous estimation procedure should therefore include the estimation of all the unknown parameters in both the functional and stochastic models. A general procedure for the parameterization and estimation of the elements of a complex stochastic model is described below.

3.1 Estimating variance-covariance components

Without loss of generality, the covariance matrix C can be parameterized as:

$$C = \sum_{i=1}^{k} \theta_i T_i \, , \qquad (12)$$

where $\theta_1, \theta_2, \ldots, \theta_k$ are the variance-covariance components of the measurements, or the unknown parameters in the covariance matrix to be estimated; k is the number of variance-covariance components; and $T_1, T_2, \ldots, T_k$ are the so-called *accompanying matrices*. According to Rao (1971), a minimum norm quadratic unbiased estimation of a linear function of $\theta_i \, (i = 1, 2, \ldots\ldots, k)$, i.e., $g_1\theta_1 + g_2\theta_2 + \ldots + g_k\theta_k$, is the quadratic function $l^T Ml$, if the matrix M is determined by solving the following matrix trace minimum problem (Rao,1971)

$$Tr\{MCMC\} = \min \, , \qquad (13)$$

subject to

$$MA = 0 \, , \qquad (14)$$

$$Tr\{MT_i\} = g_i \quad (i = 1, 2, \ldots, k) \, , \qquad (15)$$

where $Tr(\circ)$ is the Trace operator of a matrix. Based on equations (13), (14), and (15), the variance-covariance components can be estimated as:

$$\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2, ..., \hat{\theta}_k)^T = S^{-1}q \, , \tag{16}$$

where the matrix $S = \{s_{ij}\}$ with

$$s_{ij} = Tr\{PT_i PT_j\} \, , \tag{17}$$

and the vector $q = \{q_i\}$ with

$$q_i = l^T PT_i Pl \, , \tag{18}$$

and

$$P = C^{-1}[I - A(A^T C^{-1} A)^{-1} A^T C^{-1}] \, , \tag{19}$$

where I is an Identity matrix. It is noted from equations (16), (17), (18) and (19) that the estimated variance-covariance components depend on the matrix *C*, which includes the unknown variance-covariance components themselves. Therefore, an iterative process must be used. Initially, an *a priori* value of θ_i is given by θ_i^0. With equation (16), the initial estimate $\hat{\theta}^1$ is then obtained. In the $(j+1)th$ iteration, using the previous estimate $\hat{\theta}^j$ as the *a priori* values, the new estimate is:

$$\hat{\theta}^{j+1} = S^{-1}(\hat{\theta}^j) q(\hat{\theta}^j) \qquad (j = 0, 1, 2, \ldots\ldots) \, , \tag{20}$$

which is called the iterated MINQUE of θ.

It should be noted, however, that because of the lack of enough geometric information contained within the measurements not all the unknown parameters in the stochastic model can be feasibly estimated. In practice, it is very common to use a simplified stochastic model, which is assumed to be completely known, or may just include a few unknown parameters. For instance, the stochastic model described by equation (7) actually contains one unknown parameter, hence the whole structure of the model is assumed to be known in a simple form. This makes for efficient data processing. Under such circumstances, the estimation of the unknown parameter in the stochastic model is straightforward, and doesn’t even need any iteration (see equation (10)). With the assumption that the temporal correlations between epochs are absent, all the elements of the matrix

$\sum$ could be estimated. The corresponding accompanying matrices T_i have been given in Wang et al. (1998b). However, a simultaneous estimation of both the matrix $\sum$ and the temporal correlations is still a challenge.

3.2 Treatment of temporal correlations

In order to obtain a more realistic stochastic model the covariance matrix for the measurements should be designed in such a way as to adequately reflect the error structure, and should include a reasonable number of unknown parameters (that can be feasibly estimated).

It has been long recognised that the GPS measurements are temporally correlated (for example, Vanicek et al. 1985; El-Rabbany, 1994; Wang, 1998; Howind et al., 1999; Borre and Tiberius, 2000). To take such temporal correlations into account, the error specification presented by equation (5) is replaced by:

$$\begin{bmatrix} e_1(t) \\ e_2(t) \\ \cdot \\ \cdot \\ \cdot \\ e_n(t) \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} & \cdot & \cdot & \cdot & \rho_{1n} \\ \rho_{21} & \rho_{22} & \cdot & \cdot & \cdot & \rho_{2n} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \rho_{n1} & \rho_{n2} & \cdot & \cdot & \cdot & \rho_{nn} \end{bmatrix} \cdot \begin{bmatrix} e_1(t-1) \\ e_2(t-1) \\ \cdot \\ \cdot \\ \cdot \\ e_n(t-1) \end{bmatrix} + \begin{bmatrix} u_1(t) \\ u_2(t) \\ \cdot \\ \cdot \\ \cdot \\ u_n(t) \end{bmatrix} \tag{21a}$$

or

$$e(t) = R \cdot e(t-1) + u(t) \tag{21b}$$

where *u(t)* are random variables; ρ_{ii} describes the so-called temporal correlation within the DD observables of the ith satellite pair; and ρ_{ij} presents the inter-temporal correlation between the measurements of the ith and jth satellite pairs. Equation (21) is called a *first-order vector auto-regressive model*, as used by Sargan (1961). If all the inter-temporal correlations are assumed to be absent, i.e., $\rho_{ij} = 0$, equation (21) is reduced to a *first-order scalar auto-regressive model*. In most of the previous investigations on the temporal correlations of GPS measurements, it is essentially assumed that all inter-temporal correlations (between the measurements from different satellite pairs and at different epochs) are absent, and that the temporal correlations for all the satellite pairs are the same. Therefore, equation (21) represents a more general error specification.

In equation (21), the error terms $u(t)$ are temporally independent, i.e

$$E[u(t) \cdot u(v)^T] = 0 \; , \tag{22}$$

$$E[u(t) \cdot u(t)^T] = \Omega \tag{23}$$

where $t, v = 2,..., s$. Therefore, the whole covariance matrix for the error term vector u is:

$$E(u \cdot u^T) = \Omega \otimes I_s \tag{24}$$

However, due to the temporal and inter-temporal correlations, the derivation of the covariance matrix $C = E(e \cdot e^T)$ in equation (7) is complicated. Even though such a covariance matrix is available, it is difficult, if not impossible, to estimate the variance and covariance components and the (inter-) temporal correlation coefficients simultaneously. So, a two-stage estimation procedure is necessary, in which the estimation of the matrices Ω and R is essentially separated. To achieve this a matrix G is so determined that the error term vector e can be transformed, as equation (3) is transformed into the error term vector u, i.e.

$$Ge = u \tag{25}$$

and therefore, equation (3) is transformed as:

$$\bar{l} = \bar{A}x + u \tag{26}$$

where $\bar{l} = Gl$, $\bar{A} = GA$. The structure of the matrix G has been derived (Guilkey and Schmidt, 1973):a

$$G = \begin{bmatrix} G_{11} & G_{12} & . & . & . & G_{1n} \\ G_{21} & G_{22} & . & . & . & G_{2n} \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ G_{n1} & G_{n2} & . & . & . & G_{nn} \end{bmatrix}, (ns \times ns) \tag{27}$$

with

$$G_{ii} = \begin{bmatrix} b_{ii} & 0 & 0 & . & . & . & 0 & 0 \\ -r_{ii} & 1 & 0 & . & . & . & 0 & 0 \\ 0 & -r_{ii} & 1 & . & . & . & 0 & 0 \\ . & . & . & & & & . & . \\ . & . & . & & & & . & . \\ . & . & . & & & & . & . \\ 0 & 0 & 0 & . & . & . & -r_{ii} & 1 \end{bmatrix}, (s´s) \qquad (28)$$

$$G_{ij} = \begin{bmatrix} \beta_{ij} & 0 & 0 & . & . & . & 0 & 0 \\ -\rho_{ij} & 0 & 0 & . & . & . & 0 & 0 \\ 0 & -\rho_{ij} & 0 & . & . & . & 0 & 0 \\ . & . & . & & & & . & . \\ . & . & . & & & & . & . \\ . & . & . & & & & . & . \\ 0 & 0 & 0 & . & . & . & -\rho_{ij} & 0 \end{bmatrix} (i \neq j), (s´s) \qquad (29)$$

and

$$B = \begin{bmatrix} b_{11} & 0 & . & . & . & 0 \\ b_{21} & b_{22} & . & . & . & 0 \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ b_{n1} & b_{n2} & . & . & . & b_{nn} \end{bmatrix}, (n´n) \qquad (30)$$

The elements of B can be found via triangular or cholesky decomposition of Ω and Σ. The matrix B satisfies $B \Sigma B^T = \Omega$. For instance, B can be chosen as $H_1 H_2^{-1}$, where H_1 and H_2 are lower triangular matrices satisfying the conditions $\Omega = H_1 H_1^T$ and $\Sigma = H_2 H_2^T$. By using the relationship $\Sigma = R \Sigma R^T + \Omega$ (Guilkey and Schmidt, 1973), the matrix Σ can be determined from equation:

$$Vector(\Sigma) = (I - R \otimes R)^{-1} Vector(\Omega) \qquad (31)$$

where the $Vector(\bullet)$ is constructed by stacking the rows of a matrix.

It is noted that the transformed measurements $\bar{l}$ are temporally independent and have a simple stochastic model represented by equation (24). A MINQUE procedure, as discussed in section 3.1, could be used to estimate the
unknown elements of the matrix Ω. However, the determination of the transformation matrix G relies on the elements (ρ_{ij}) of R, which are unknown and need to be estimated separately.

Since the true values of the measurement errors are unknown, the estimation of (inter-) temporal correlation coefficients, or the elements (ρ_{ij}) of R, has to be based on the residuals ($\hat{e}$) of the original DD observables l. Based on equations (21), one obtains:

$$\begin{bmatrix} \hat{e}_i(2) \\ \hat{e}_i(3) \\ . \\ . \\ . \\ \hat{e}_i(s) \end{bmatrix} = \begin{bmatrix} \hat{e}(1)^T \\ \hat{e}(2)^T \\ . \\ . \\ . \\ \hat{e}(s-1)^T \end{bmatrix} r_i + \begin{bmatrix} u_i(2) \\ u_i(3) \\ . \\ . \\ . \\ u_i(s) \end{bmatrix} \qquad i = 1, 2, ..., n \tag{32a}$$

or

$$E_{2i} = E_1 r_i + u_i \tag{32b}$$

where both E_{2i} are an $(s-1)\times 1$ vector; E_1 is an $(s-1)\times n$ matrix; r_i is an $n\times 1$ vector representing the ith row of matrix R. The unknown vector r_i can therefore be estimated by applying the least-squares principle to equation (32), or all the elements of R are estimated together as:

$$Vector(\hat{R}) = [E_a^T(\Omega^{-1}\otimes I_{s-1})E_a]^{-1} E_a^T(\Omega^{-1}\otimes I_{s-1})E_b \tag{33}$$

with

$$E_a = \begin{bmatrix} E_1 \\ E_1 \\ . \\ . \\ . \\ E_1 \end{bmatrix} \quad \text{and} \quad E_b = \begin{bmatrix} E_{21} \\ E_{22} \\ . \\ . \\ . \\ E_{2n} \end{bmatrix}$$

Because the estimation of the residuals depends on the covariance matrix, an iterative estimation procedure is required.

3.3 An iterative stochastic modelling procedure

Based on the above theoretical analysis, an iterative procedure for the estimation of the (inter-) temporal correlation matrix R and the covariance matrix Ω is summarised as follows:

Preparatory steps:

(1) use the standard stochastic model represented by equation (7)
(2) obtain estimates of the unknown parameters and residuals using equations (8) and (9);
(3) estimate the covariance matrix ($\hat{\Sigma} = \hat{\Omega}$) with equation (10);

Iterative steps:

(4) estimate the temporal correlation matrix $\hat{R}$ using equation (33);
(5) construct the transform matrix G using equations (27), (28), (29) and (30) with the matrices $\hat{R}$ and $\hat{\Omega}$;
(6) estimate the covariance matrix ($\hat{\Omega}$) for the transformed measurements $\bar{l}$ using the MINQUE procedure;
(7) obtain estimates of the unknown parameters from $\hat{x} = [\bar{A}^T(\hat{\Omega}^{-1} \otimes I_s)\bar{A}]^{-1}\bar{A}^T(\hat{\Omega}^{-1} \otimes I_s)\bar{l}$;
(8) obtain the residuals $\hat{e}$ from equation (9) using the estimated unknown parameters $\hat{x}$;
(9) check the variations of the estimated elements of the matrices $\hat{R}$ and $\hat{\Omega}$;
(10) Stop iteration if sufficient accuracy (say, 0.001mm in the baseline components) is achieved. Otherwise, go back to step (4).

4 Experimental Results and Analysis

4.1 Description of the data sets

To demonstrate the impact of various stochastic modelling procedures on GPS relative positioning, three static GPS baseline data sets (see Table 1) were analysed. For all the data sets, the data interval is 15 second and the session length is 30 minutes. In the data processing, only L1 frequency data were used.

It should be noted that in the case of the Ashtech data set, two receivers were mounted on pillars that are part of a first-order terrestrial survey network. The known baseline length between the two pillars is 215.929 ± 0.001 m, which will be used as a ground truth to check the results obtained using the various stochastic modelling procedures (Section 4.3).

4.2 Data processing methods

All the data sets were processed using the stochastic modelling methods:

A. The standard procedure with the stochastic model expressed by equation (7), assuming that temporal correlations are absent $R = 0$.

B. A modified standard procedure with $R_{ij} = 0$ and $R_{ii} = R_{jj}$ $(i \neq j)$, i.e., assuming that R_{ii} is the same for every satellite pair and follows an exponential function defined by El-Rabbany (1994).

C. A two-stage method with $R_{ij} = 0$ and $R_{ii} \neq R_{jj}$ $(i \neq j)$ *(a first-order scalar auto-regressive model)*, and applying the MINQUE procedure to estimate a variance-covariance matrix for transformed measurements.

D. A two-stage method with $R_{ij} \neq 0$ $(i \neq j)$ *(a first-order vector auto-regressive model)*, applying the MINQUE procedure to estimate a variance-covariance matrix for the transformed measurements.

4.3 Analysis of results

The DD residuals for each data set have been presented in Figures 1 to 3, which show the time series of the DD residuals obtained from the baselines B15M, B215M and B13KM respectively. Because the residuals obtained by methods B, C and D showed similar trends, for simplicity of discussion only the residuals obtained by method D are compared with those obtained from method A. The ‘heavy’ lines represent the residuals obtained from method A, while the ‘light’ lines show the residuals obtained from method D. Among these stochastic models, the preferred one will produce randomised residuals. Therefore, methods C and D are better than methods A and B. In general, if more parameters are included within the functional model, there is a better fit of the model with the data. Hence, in such a case, the resulting residuals may also become more random. However, not all of the parameters can be reliably estimated in the solution, as there may not be sufficient information in the measurements. The results in figures 1-3 do indicate that unmodelled (residual) errors can be accommodated by stochastic modelling.

It can be seen from these figures that, for most of the satellite pairs, the standard processing results in residuals that exhibit significant systematic errors. This is further confirmed by the temporal correlation coefficients listed in Table 2. The temporal coefficients in Table 2 are computed by applying the Durbin-Watson statistic

(Durbin and Watson, 1950) to DD residuals. For method A, these DD residuals are associated with the original measurements. For methods B, C and D, the DD residuals are associated with the transformed measurements. In the case of method D, the residuals for one satellite pair may include a minor component from other satellite pairs due to the inter-temporal correlations. The time series of DD residuals in Figure 2 show some significant multipath errors for satellite pairs PRN 2-7 and 2-19 (PRNs 7 and 19 have elevation at 15 and 19 degrees, respectively). With reference to Figures 1 to 3, it is clearly evident that the systematic errors of the transformed measurements are much smaller than those of the original measurements. This is because the residuals for the transformed measurements are more random than those of the original measurements.

It is noted that the temporal correlation coefficients from method B are large and negative. Table 2 indicates that fixing the temporal correlation coefficients to the same value for all the satellite pairs might be inappropriate in reality. As expected, it is clearly seen from Table 2 that the estimated temporal coefficients obtained by methods C and D are closer to zero than those obtained from methods A and B. The residuals obtained by methods C and D are therefore essentially random, which indicates that the temporal correlations have been taken into account in the measurement transformation step.

It has been shown in, for example, Teunissen (1997) and Wang et al. (1998b), that the stochastic models have a significant influence on ambiguity resolution. The discrimination test is one of the critical steps. Both the classical F-ratio and a recently developed statistic Ws (Wang et al. 1998a) are used here. The larger the values of these statistics, the more reliable the ambiguity resolution. For method B, all three baselines have small F-ratio values (Table 3). In the case of baseline B15M, both methods C and D produce larger F- and Ws-ratio than methods A and B. However, in the case of baselines B215M and B13KM, the contrary results were obtained. These phenomena might be linked to the systematic errors existing in the measurements. But, in view of the fact that methods C and D generate random residuals for all the baselines, the F-ratio and W-ratio statistics obtained by methods C and D could be considered to be more realistic than those obtained by methods A and B.

The estimated baseline components and their *a-posteriori* standard deviations are presented in Table 4. Over all, method B produces larger standard deviations than the other methods, although the other methods show no significant difference in this regard. These results also indicate that there is generally no significant difference in the horizontal components. However, it is important to note that in the case of baselines B215M and B13KM, the differences in estimated height components between methods A and C (or D) can be as large as 10 mm. This is a significant difference for high-precision applications, and thus, a realistic stochastic model is critical for such applications. For the baseline B215M, the estimated baseline lengths using methods C and D are much closer to the known baseline length than using methods A and B.

5. CONCLUDING REMARKS

A realistic stochastic model for GPS measurements is critical for reliable ambiguity resolution and baseline component estimation. With three static baseline data sets, some aspects of the misspecification in the stochastic model were analysed in detail. It was shown that the GPS measurements have a heteroscedastic, space- and time-correlated error structure, and that any misspecification in the stochastic model may have a significant influence GPS data processing.

After reviewing the existing methods, an iterative stochastic modelling procedure has been proposed to directly estimate the time correlation coefficients, and the time-independent variance and covariance components of the GPS measurements. In the proposed procedure, the commonly used stochastic model is first used to estimate approximate values of the temporal correlation coefficients. Based on the estimated time correlation coefficients, the original DD observables are then transformed into a new set of measurements. These transformed measurements are free of time correlations, and thus have a block diagonal covariance matrix. The covariance matrix for the new measurements can be estimated using the MINQUE method. An advantage of the transformed DD carrier phases is that the effects of systematic errors are largely eliminated, and thus the resulting residuals may be considered random. By removing the systematic errors in the measurements, as expected, the reliability of the estimated positioning results is improved. In addition, the quality of ambiguity resolution can be more realistically indicated.

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