Modeling and Geometry Design for Pseudolite Augmented Airborne DGPS

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BIOGRAPHY

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ABSTRACT

Carrier phase DGPS systems have been increasingly used in airborne surveying. However, the accuracy and availability of GPS positioning cannot meet the stringent requirements of large-scale photogrammetry. Ground-based pseudolites can strengthen the measurement geometry for airborne GPS systems, and improve ambiguity resolution. As a result, positioning accuracy and reliability can be improved, especially in the vertical component.

In order to effectively augment DGPS aided airborne surveying with pseudolites, carrier phase measurements of pseudolites need to be employed in the positioning process. However, due to the comparatively small separations between pseudolites and receivers, some challenging issues have to be investigated, such as geometry design, nonlinearity, tropospheric delay and pseudolite location errors.

Geometry design is extremely important for pseudolite augmented GPS positioning system. Optimally located pseudolite and reference receiver can significantly improve the geometric strength of positioning solutions and thus reduce the effects of nonlinearity and pseudolite location error. The optimal locations of pseudolite and reference receiver are proposed based on analyzing these issues and the simulation results in airborne surveying scenarios.

Nonlinearity is a challenging issue in pseudolite augmented DGPS. The nonlinear geometry bias in processing pseudolite measurements can be effectively eliminated by Projected Single Difference strategy, which can also be used in processing GPS measurements to improve the positioning accuracy. Several pseudolite tropospheric delay models are introduced and evaluated, and optimal models corresponding to different applications are proposed. Simulation and real data test results have shown the effectiveness of the proposed methods.

INTRODUCTION

GPS has been widely used for airborne positioning and navigation applications. However, in some applications, such as aircraft precision landing and large scale airborne surveying, the requirements of positioning accuracy, availability and integrity cannot be satisfied by GPS alone. Due to the limited constellation of GPS satellites and their geometric distribution, the accuracy of GPS positioning in the vertical components is about three times worse than that of the horizontal components.

Pseudolites (PLs) transmit GPS-like signals. Ground based PLs can be used to augment DGPS system for precise positioning and navigation. With the additional ranging signals from PLs, positioning solutions can be less sensitive to the failure of GPS satellite signals and more accurate due to the improvement of geometry strength. PL augmented DPGS has been successfully applied in Local Area Augmentation System (LAAS) for precision aircraft landings (Barltrop et al., 1996; Hein et al., 1997). PL augmented DGPS can also be used in large scale airborne remote sensing systems, which need very accurate positioning solutions.

This paper investigates some issues of PL augmented DGPS in airborne surveying scenario, including system geometry design, nonlinearity correction, tropospheric delay and PL location errors (as orbit error for GPS satellite) analysis. As PLs are comparatively close to receivers, the unit vectors from a PL to the reference and rover receivers can be significantly different. PL-location error and tropospheric delay cannot be effectively mitigated in the differencing procedure. The effects of nonlinearity and PL-location error are significant and sensitive to PL and receivers' geometry.

PL signals only propagate through the lower troposphere, where it is very difficult to model the signal delay due to spatial variations in atmospheric pressure, temperature and humidity. GPS satellite tropospheric delay models cannot be directly used for PL. Several models are introduced and evaluated in this paper for accurately estimating PL tropospheric delay. Unlike the vertical variation of tropospheric delay, horizontal variation is more random and difficult to model. Differencing procedure can mitigate horizontal variation with the help of proper geometry design.

Nonlinear geometry bias exists in single difference (SD) when the unit vectors from the reference and user receivers to the satellite or PL are unparallel, as shown in Figure 1. A Projected Single Difference (PSD) approach was introduced recently (Zhang and Bartone, 2003) to deal with this problem, which is superior to previous methods. Optimized expression is derived to calculate the direction of project vector, and the advantages of applying PSD in PL augmented airborne DGPS are demonstrated.

Geometry design is to optimally locate PLs and reference receiver in order to strengthen the GPS and PL geometry and reduce systemic errors, and hence to improve reliability and accuracy of the final positioning solutions. Reducing dilution of precision factor, the effects of PL location error and horizontal variation of PL tropospheric delay are the goal for geometry design, simultaneously considering practical issues, such as PL signal near-far problem, flight area and directions etc. The following sections discuss these issues in detail with the help of simulation and flight test.

NONLINEARITY

In relative GPS positioning, a referencing receiver and a user receiver are used to mitigate common systemic errors and get the integer ambiguity resolution. Single difference eliminates satellite clock error and mitigates most of troposphere and ionosphere errors. Double difference (DD) eliminates the receiver clock bias. The DD observations array $\mathbf{D_{dd}}$ can be used to solve baseline coordinate vector $\boldsymbol{b}$ by Equation 1:

$$\mathbf{D_{dd}} = \mathbf{H}\,\boldsymbol{b} + \boldsymbol{e} \qquad (1)$$

where $\mathbf{H}$ is the geometry matrix of differenced unit vectors and $\mathbf{e}$ is measurement error vector, which is to be minimized.

The **H** matrix is constructed by differenced unit vectors in the direction of the two range sources from the reference receiver assuming parallel lines, which is an approximation for short baseline DGPS. The **H** matrix provides the comprehensive geometry information between receivers and the GPS satellites. The faster the geometry changes, the greater the changing rate of elements in the **H** matrix, the more significant the performance of DGPS solution improves in terms of efficiency, accuracy and availability.

PL augmented GPS system can significantly increase the dynamic geometry. At the same time, a nonlinear component is introduced in SD because the unit vector from the reference receiver to the PL ($\boldsymbol{e_r}$) is not the same as the one from the user receiver ($\boldsymbol{e_u}$). Figure 1 describes the nonlinear problem in SD. The vectors $\boldsymbol{e_r}$ and $\boldsymbol{e_u}$, are unparallel when the distance between GPS/PL and the receivers cannot be assumed infinity comparing to the length of baseline $\|\boldsymbol{b}\|$. The nonlinear component $\mathbf{X_{ru}}$ equals to the difference of $\mathrm{SD}_{\mathbf{ru}}$ and $\mathrm{PSD}_{\mathbf{ru}}$.

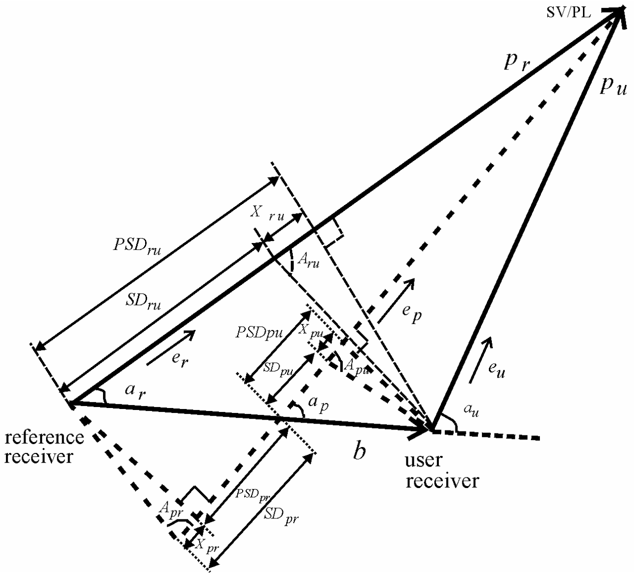

Figure 1 Nonlinearity problem in single difference

In normal DGPS, SD is projected in the direction of $\boldsymbol{e}_\mathbf{r}$ and the unit vector used in the **H** matrix is also from $\boldsymbol{e}_\mathbf{r}$. Geometry correction is needed to remove the nonlinear component $\mathrm{X}_{\mathbf{ru}}$ from the SD. $\mathrm{X}_{\mathbf{ru}}$ is decided by the length of baseline $\|\boldsymbol{b}\|$, angles $\mathrm{a}_\mathbf{r}$ and $\mathrm{A}_{\mathbf{ru}}$ as in Equation 2.

$$\boldsymbol{b}\cdot\boldsymbol{e_r} = \mathrm{PSD}_{\mathbf{ru}} = \mathrm{SD}_{\mathbf{ru}} + \mathrm{X}_{\mathbf{ru}}$$

and (2)

$$X_{ru} = \frac{\|b\|\cdot\sin a_r}{\tan A_{ru}}$$

As shown in Equation 2, initial baseline estimation error will propagate into nonlinear component $\mathrm{X}_{\mathbf{ru}}$ and induce geometry correction error. This may lead to incorrect carrier phase ambiguity estimates. Moreover, the reference unit vector $\boldsymbol{e_r}$ does not change over time for PL augmentation because the PL and reference receiver are fixed. Therefore the normal DGPS baseline solution cannot take the advantage of geometry change when $\boldsymbol{e_r}$ is used in the **H** matrix in the Equation 1.

The PSD method projects both $?_r$ and $?_u$ on a new direction $\boldsymbol{e_p}$, and so there are two nonlinear components X_{pu} and X_{pr} apply to the two single differences SD_{pu} and SD_{pr}, as shown in Figure 1. The nonlinear components change when the SD is projected in different directions. The $\boldsymbol{e_p}$ can be found, in the direction from a point on the baseline $\boldsymbol{b}$ to the SV/PL, to let X_{pu} and X_{pr} be the same. Therefore the combined geometry correction goes to zero and the nonlinear component is removed.

$$\begin{aligned} \boldsymbol{b}\cdot \boldsymbol{e}_p &= PSD_{pu} + PSD_{pr} \\ &= SD_{pu} + X_{pu} + SD_{pr} - X_{pr} \qquad (3) \\ &= SD_{pu} + SD_{pr} \\ &= SD_{ru} \end{aligned}$$

Based on Equation 3 and the geometric relationships in Figure 1, the following Equation 4 can be derived to calculate the angle a_p:

$$\begin{aligned} \cos a_p &= \frac{\sin a_u - \sin a_r}{\sin(a_u - a_r)} \\ &= \cos a_r + \sin a_r \cdot \tan(\Delta a/2) \qquad (4) \\ \text{where } \Delta a &= a_u - a_r \end{aligned}$$

According to Equation 4, angle a_p equals to a_r when $?a$ is zero, and a_p deviates from a_r when $?a$ becomes large. The baseline $\boldsymbol{b}$ is not directly involved in Equation 4, so the calculation error of a_p is not sensitive to the initial baseline estimation error.

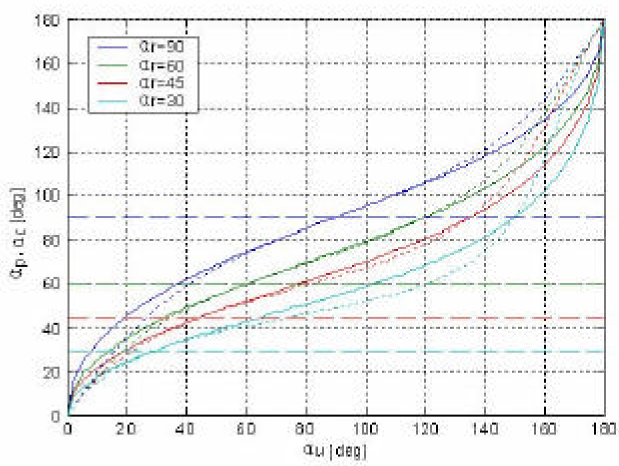

Figure 2 The change of a_p in PSD against a_u with different a_r.

Figure 2 describes the changes of angle a_p in PSD against a_u with different a_r. The horizontal dashed lines are the selected values of a_r. The dashed curve lines are the calculated angle a_c expressing a project vector from the center point of the baseline to PL, which is usually used in long baseline DGPS. The figure shows that a_c ˜ a_p when a_u ˜ a_r and a_p is largely depending on the change of a_u.

There is a positioning bias caused by nonlinearity using baseline center point approximation when $?a$ is not ignorable. The bias is about one meter for a 200km baseline.

In the case of airborne surveying scenario, showing in Figure 3, two PL locations are selected. PL2 is located inside and PL1 outside of a 4km x 4km surveying area. The reference receiver is at and the start point of the user receiver is above the middle of the surveying field. The aircraft's trajectory is nearly a square in a horizontal plane at 450 meters' height.

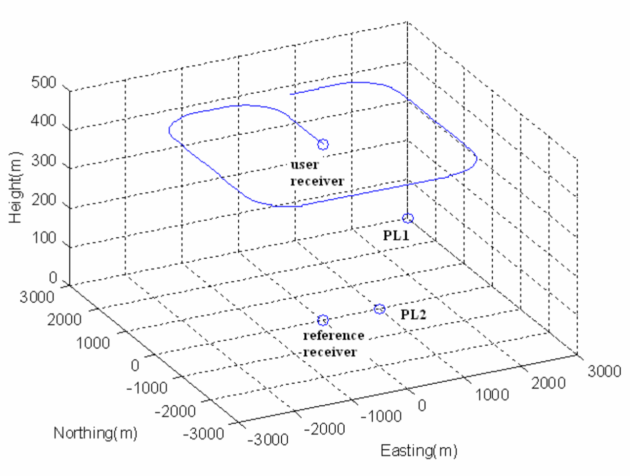

Figure 3 PL augmented airborne surveying scenario

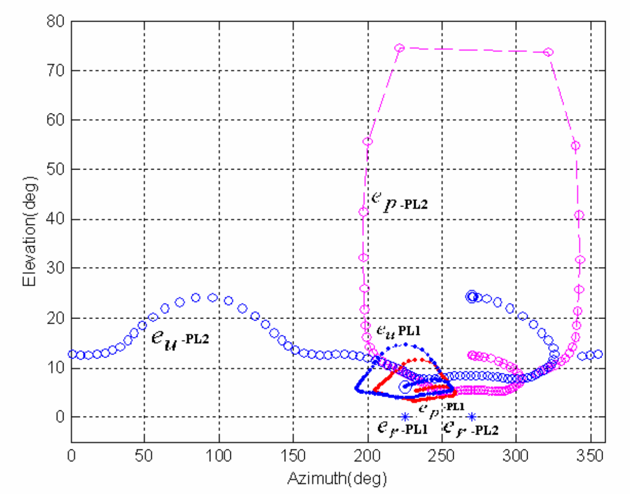

Figure 4 The directions of $\boldsymbol{e_r}$, $\boldsymbol{e_p}$ and $\boldsymbol{e_u}$.

The changes of unit vectors $\boldsymbol{e_r}$, $\boldsymbol{e_p}$ and $\boldsymbol{e_u}$ with regard to the airplane trajectory are plotted in Figure 4. The unit vector $\boldsymbol{e_r}$ keeps unchanged because both the reference receiver and PL are stationary. The vectors $\boldsymbol{e_u}$ and $\boldsymbol{e_p}$ calculated from Equation 4 change much as the aircraft flying along the trajectory, especially when PL is located inside the surveying area. Therefore the projected DGPS (PDGPS) baseline solution can take the advantage of geometry change of PL using $\boldsymbol{e_p}$ in the **H** matrix in Equation1. In normal DGPS, however, unit vector $\boldsymbol{e_r}$ is used in the **H** matrix in Equation 1 and it is constant for the PL.

By introducing the project unit vector $\boldsymbol{e}_{\mathbf{p}}$ in the PSD**,** nonlinear component is eliminated. Then the normal double difference procedure can be applied without introducing geometry correction error or losing dynamic geometry advantage. PDGPS can remove the nonlinear bias and keep the integer ambiguities of carrier phase measurements, and consequently improve the accuracy of carrier phase DGPS positioning. Noticing Equation 4 is simple and no any approximation, it is sensible to apply the PSD for both PL and GPS measurements in GPS/PL aided airborne surveying because centimeter level accuracy is expected.

TROPSPHERIC DELAY

As signals from a PL to a reference receiver and to a rover receiver pass through very different parts of troposphere, common GPS tropospheric delay models are not sufficient for PL positioning. Tropospheric delay can be the largest error source of PL measurements as PL signal propagates through the lower troposphere where the variability of water vapor is very difficult to model. Accurate models of the tropospheric effect are needed for PL.

In general, the truth model of tropospheric delay ?^{trop} is a function of temperature **T**, atmospheric pressure **p** and relative humidity **r** etc. atmospheric parameters along the signal path **s** (Hofmann-Wellenhof et al., 2000). N in Equation 5 is the refractivity of troposphere.

$$\Delta^{trop} = 10^{-6} \int N\left(T, p, r\right) ds \qquad (5)$$

As atmospheric parameters are usually measured at the reference station, there are biases to estimate them along the path of signal propagation. Several PL tropospheric delay models have been introduced, such as RTCA model (RTCA, 2000) and its modification (Biberger et al., 2003) regarding LAAS for precise aircraft approaching and landing. The Bouska model (Bouska and Raquet, 2003) is derived from the Hopfield model. These models compute the tropospheric delay as a function of the local refractivity along the PL signal path.

Another method is introduced using single-differenced GPS tropospheric delay models (Wang et al., 2004). Suppose there is a GPS satellite in the same line of a PL and a receiver. Tropospheric delays can be calculated from the receiver and the PL to the GPS satellite using the well-known NMF and Saastamoinen models. The tropospheric delay from the PL to the receiver is the difference of the two values. As the models of GPS tropospheric delay are relatively better developed and can reach high accuracy, it is reasonable to derive PL tropospheric delay models from them. However, the performance of GPS tropospheric delay models degrades at very low elevation angles. Models derived from them could have big bias in a low elevation angle.

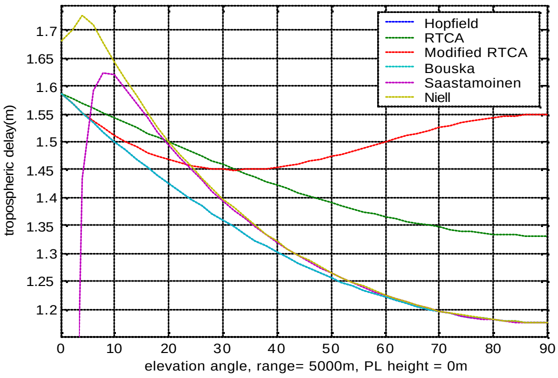

Figure 5 Comparing PL tropospheric delay models with different elevation angles

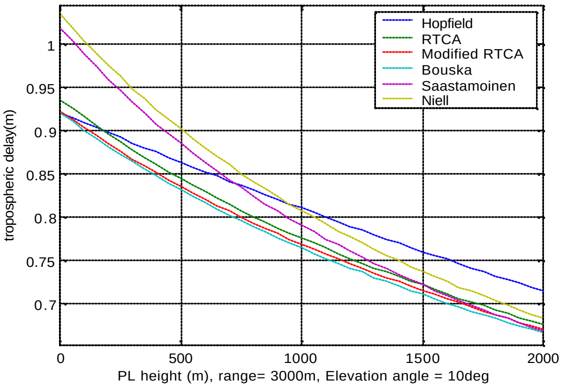

Figure 6 Comparing PL tropospheric delay models with different reference altitude

Figure 5 shows the tropospheric delays calculated from different models in a range of 5km with reference receiver at sea level. The elevation angle changes from 0 to 90 degrees. The values given by the Niell model with elevation angles less than 4 degrees and by Saastamoinen model with elevation angle less than 10 degrees should be ignored. The difference of tropospheric delays estimated with different models can reach more than 30cm in a range of 5km. Figure 6 shows the tropospheric delays calculated from different models in a range of 3km with a 10 degree elevation angle. The reference altitude changes from 0 to 2000 meters.

It can be concluded from the simulation results that the differences of tropospheric delays estimated with different models are observable. The TRCA model can be used in the applications with small altitude difference, such as aircraft landing or land-based applications, but not suitable for applications with large altitude difference, such as precise airborne surveying. Single differenced Niell and Saastamoinen models are reliable at high elevation angle but not at low elevation angle. The Hopfield and Bouska models perform relatively stable over the whole range of elevation angle though there is a bias between them.

The GPS/PL carrier phase measurements from a flight test were processed. The DD residuals are used to analyze the performance of different models of PL tropospheric delay. Figure 7 shows the carrier phase DD results by applying the PL tropospheric delay models introduced above. It is found that the result of the NMF model is the best one among all the models tested. The results of the Saastamoinen model vary violently during most epochs as the elevation angles from the PL to the rover receiver are less than 5 degrees during most epochs. The single-differenced method is very effective to estimate PL tropospheric delay by employing well developed GPS tropospheric models.

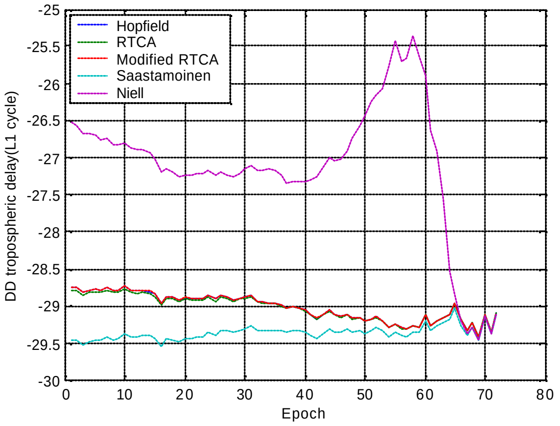

Figure 7 Double differenced carrier phase results with differennt tropospheric delays models

Flight test results confirm some of the conclusions from the simulation results. The single-differenced method is effective to estimate PL tropospheric delay by employing GPS tropospheric delay models. It is found that the result of the NMF model is the best one among all the models tested even if the elevation angle is very small in the flight test. However, as the flight test data does not cover the range used in the simulation, the other conclusions from the simulation results should be further tested.

GEOMETRY DESIGN

Geometry design is very important for PL augmented DGPS positioning systems. Optimally located PL and reference receiver can significantly improve the geometric strength of positioning solutions; reduce the effects of nonlinearity and pseudolite location error, and hence have a large impact on the reliability and accuracy of positioning solutions.

Dilution of Precision

The geometry of the visible GPS satellites and PLs is an important factor in achieving high quality positioning results. A measure of the geometry is the dilution of precision (DOP), a factor depending on the geometry of GPS satellites and PLs relative to a receiver, which generally can be calculated from the inverse of the normal equation matrix of the solution. GPS/PL positioning accuracy is decided by the product of factor DOP and measurement accuracy. DOP is used for point positioning and relative DOP (RDOP) is used for baseline processing. Consequently GPS positioning accuracy can be improved by both improving GPS/PL measurement accuracy and reducing the factor DOP by augmenting the geometry of SV/PL distribution.

By optimizing the PL location with respect to a receiver, PL can significantly improve the geometrical strength of GPS positioning solutions, particularly for the height component (Wang, 2002; Wang et al., 2001). Therefore one aim of geometric optimization of PL augmented positioning system is to select locations for PL that will minimize DOP.

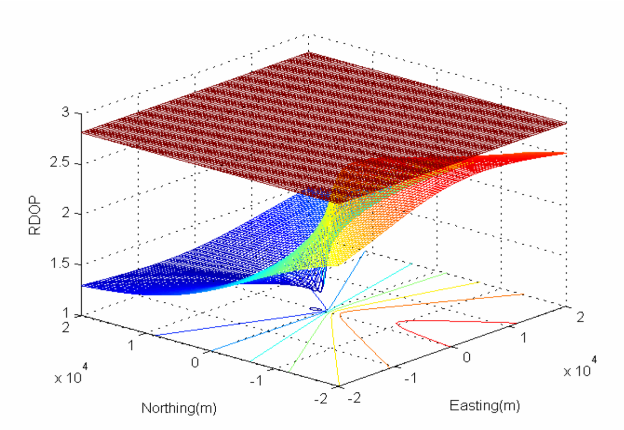

Figure 8 RDOP changes with PL location

Figure 8 reveals how PL location affects RDOP. A PL was moved horizontally from -20,000m to 20,000m with 500m intervals, and the GPS satellites distribution was fixed. The user receiver is fixed at the middle of the area and the altitudes of the receiver and the PL was at 500m and 5m respectively, which means the PL elevation angle is negative. The upper mesh in Figure 8 shows the RDOP values without PL augmentation and the lower one represents RDOP changes with the PL location varying across the 500m grids. The RDOP values vary between 1.2 and 2.8, depending on the PL location. Generally speaking, an appropriately located PL will reduce RDOP significantly and hence improve the positioning accuracy. As surveying regions are predefined and the GPS satellite constellation is predictable, it is convenient to design the PL and reference receiver location to minimize RDOP.

Pseudolite location error

In relative GPS positioning, the impact of the orbit errors on baseline length was estimated approximately using the 'rule of thumb' formula (Wells et al., 1987). In PL relative positioning, however, the impact of PL location error is different. From the Figure 1, the baseline vector ***b*** can be expressed by the Equation 5:

$$\boldsymbol{b} = ?_{\mathbf{r}} - ?_{\mathbf{u}} \tag{5}$$

With d||***b***|| and d||***p***|| indicate the baseline error and PL location error respectively, the Equation 6 can be derived, noticing that $\mathbf{e}_r = ?_r / \|?_r\|$ and $\mathbf{e}_u = ?_u / \|?_u\|$.

$$d\,\|b\| \big/ d\,\|p\| = \|e_r - e_u\| \qquad (6)$$

The Equation 6 indicates that baseline measurement error d||***b***|| is geometry-dependent. The maximum of the left side of Equation 6 is 2 when $\mathbf{e}_r$ and $\mathbf{e}_u$ is antiparallel and the minimum of it is 0 when $\mathbf{e}_r$ and $\mathbf{e}_u$ is parallel. The factor **d||*b*||/d||*p*||** changes between 0 and 2 according to the relative locations between PL and the two receivers. The 'rule of thumb' is just an approximation of the Equation 6 when $\mathbf{e}_r$ and $\mathbf{e}_u$ is almost parallel, which is true for relative GPS positioning.

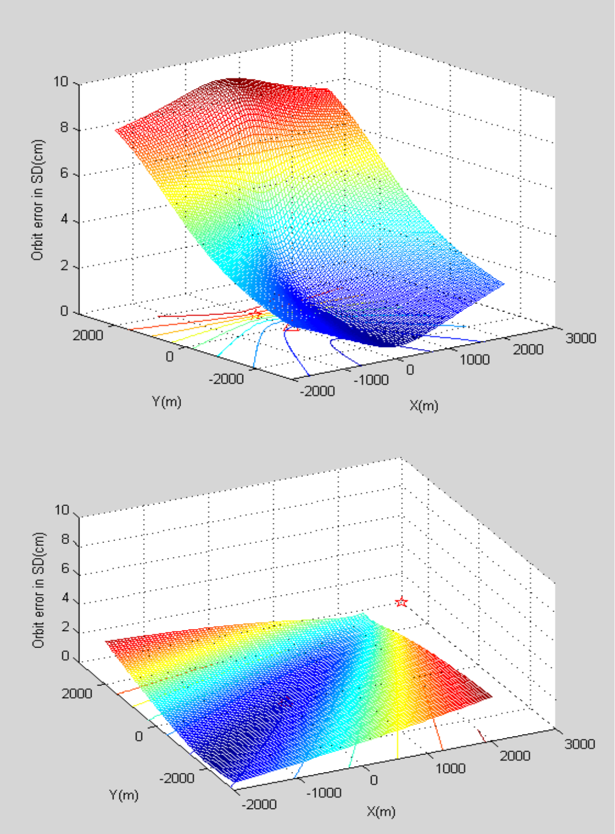

Figure 9 Baseline errors when PL is inside (top) and outside (bottom) the surveying area.

Simulations were conducted for analyzing the airborne surveying scenario shown in Figure 3. The two patterns in Figure 9 show the baseline errors measured by PL with 5 centimeter PL location error. The errors are small and flat when the PL is located outside the airborne surveying area but much higher and more discrepant when the PL is located inside the area. It can be seen clearly that the influence of PL location error varies significantly with different relative locations. In the worst case, the SD procedure doubles the size of the PL location error. Therefore, if the PL is improperly located, the PL location error can bias PL carrier phase SD observations in the same magnitude of the PL location error.

Practical consideration

More issues need to be considered during the geometry design for PL augmented airborne DGPS positioning system.

Horizontal variation of PL tropospheric delay is nearly random and very difficult to model. As has been mentioned, the variability of the water vapor is of much concern in accurate GPS applications. The water vapor exists mostly in the lower 5 km of the troposphere. Its distribution may depend on azimuth primarily due to terrain and wine effects. In PL augmented GPS system, single differencing procedure can mitigate it if a proper geometry design lets the horizontal projection of the signal paths from the PL to the reference receiver and to the user receiver are as similar as possible.

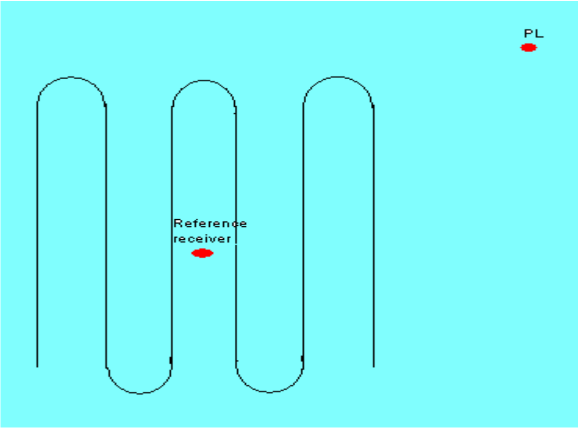

Figure 10 Optimal PL and receiver locations and trajectory design for airborne surveying

At the same time, the PL signal near-far problem has to be considered in the geometry design. As the received signal power is inversely proportional to the square of the distance between a PL to a receiver, there is significant signal strength variation due to the relative change in the distance between the PL and receivers. The PL signal may be below the detection threshold of GPS receivers if the distance is beyond the far-limit, whereas GPS receivers may be overwhelmed by the strength of a PL transmission and jam out the signals from GPS satellites. The PL should be properly located to avoid the near-far problem for both reference receiver and user receiver.

A comprehensive analysis should be conducted for the geometry design of GPS/PL aided airborne surveying. As shown in Figure 10, locating a reference receiver at the centre of a surveying area and the PL outside it at proper direction and distance can minimize the impacts of PL location error and the tropospheric delay modeling error, and reduce the RDOP value at the same time.

CONCLUDING REMARKS

The nonlinear problem, PL tropospheric delay and geometry design of PL augmented airborne DGPS positioning have been discussed in this paper. By introducing the project unit vector $\boldsymbol{e}_{\mathrm{p}}$ in the PSD**,** nonlinear component is eliminated without introducing geometry correction error or losing dynamic geometry advantage. Then the normal double difference procedure can be applied and PDGPS can remove nonlinear bias and keep integer ambiguities of carrier phase measurements, and consequently improve the accuracy of DGPS positioning.

An optimized tropospheric delay model is also critical for accurate PL measurements though the locations of PL and reference receiver also affect the troposphere delay modeling error. The advantages and limitations of several PL tropospheric delay models have been investigated in detail by analyzing simulation results. Flight test data were processed to verify the results. The single-differenced NMF model has been proposed to be the best model for GPS/PL aided airborne surveying scenario. Comprehensive analysis have been conducted for the geometry design of GPS/PL aided airborne surveying, considering the impacts of PL location error and the tropospheric delay modeling error, and reducing the RDOP value at the same time.

ACKNOWLEDGEMENTS

The author is grateful to Dr. Jinling Wang for his guidance and valuable advice on this study. This research is supported by an ARC (Australian Research Council) research project on 'Integration of GPS/Pseudolite/INS to Geo-reference Airborne Surveying and Mapping Sensors and cooperated with QASCO SURVERS PTY LTD.

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