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Pseudolite Augmentation for GPS Aided Aerial Photogrammetry: An Analysis of Systematic Errors

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Abstract

GPS has been widely used as a geo-referencing tool in aerial surveying. However, the accuracy and availability of GPS positioning cannot meet the stringent requirements of large-scale photogrammetry. Ground-based pseudolites can strengthen measurement geometry for GPS based airborne geo-referencing systems. As a result, positioning accuracy and reliability can be improved, especially in the vertical component.

However, as pseudolites are comparatively close to receivers, some challenging issues in systematic error analyses and modeling need to be further investigated. In this paper, the major systematic errors related to pseudolites, such as tropospheric delay, multipath and pseudolite location errors are analysed, and their impacts on the performance of an integrated GPS/Pseudolite airborne geo-reference system are presented.

1. INTRODUCTION

GPS has been increasingly used for airborne positioning and navigation applications. However, in some applications, such as aircraft precision landing and airborne surveying, the requirements of positioning accuracy, availability and integrity cannot be satisfied by GPS alone (Hein et al. 1997; Lee et al. 2002). Due to the limited constellation of GPS satellites and their geometric distribution, the accuracy of GPS positioning in the vertical components is about three times worse than that of the horizontal components.

This problem can be addressed by the inclusion of additional ranging signals transmitted from ground-based pseudolites (PLs), which transmit GPS-like signals. The pseudolites augmented GPS system will have improved performance because the availability and geometry of GPS can be strengthened significantly.

Figure 1 illustrates the concept of pseudolite augmentation for an airborne surveying application using PL augmented relative GPS kinematic positioning technology. Two antennas are installed on the aircraft fuselage, one on the top and the other underneath. The antenna beneath the fuselage facing ground is used to receive the signal from a pseudolite on the ground.

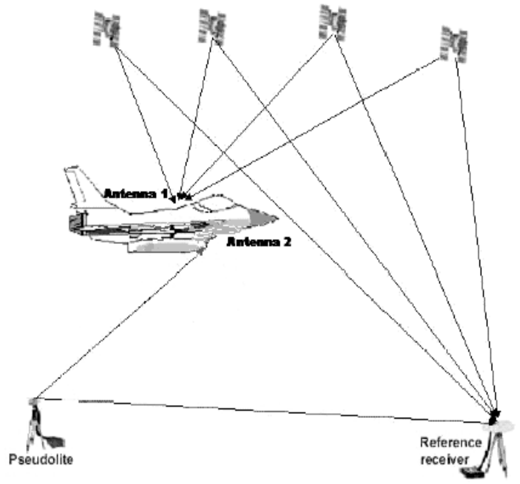


Figure 1 *Pseudolite augmentation for airborne positioning application*

It is well known that GPS positioning accuracy can be described as the product of the ranging error and the dilution of precision (DOP), the effect of satellite configuration geometry (Rizos 1997):

$$\textbf{\textit{Positioning Error}} = \textbf{\textit{Dilution of Precision}} \times \textbf{\textit{Ranging Error}} \qquad (1)$$

where the Dilution of Precision is a class of multipliers, such as HDOP (Horizontal DOP), VDOP(Vertical DOP), or other similar factors. DOP is for single point positioning and RDOP is for baseline processing.

Consequently there are two ways to improve GPS positioning accuracy: augmentation of the GPS observation accuracy and the geometry of the satellites' distribution. Pseudolites can be used to improve the geometrical strength of positioning solutions, particularly for the height component (Wang et al. 2001; Wang 2002) by optimizing the pseudolites location with respect to the receiver. Geometric optimization refers to finding locations for the pseudolite transmitters that will minimize DOP.

On the other hand, it is required that the accuracy of PL measurements is as good as those of GPS measurements. Due to the comparatively small separation between pseudolites and receivers, there are some challenging issues in modelling and geometry design that need to be dealt with to improve PL measurement accuracy, such as pseudolite tropospheric delays, multipath and pseudolite location errors. These issues are investigated in following sections.

2. GEOMETRY DESIGN

Appropriately located pseudolite(s) can strengthen the GPS satellite geometry and signal availability significantly, and hence do have a large impact on the reliability and accuracy of positioning solutions. The best pseudolite location(s) at a certain time depends on satellite positions and rover receiver position.

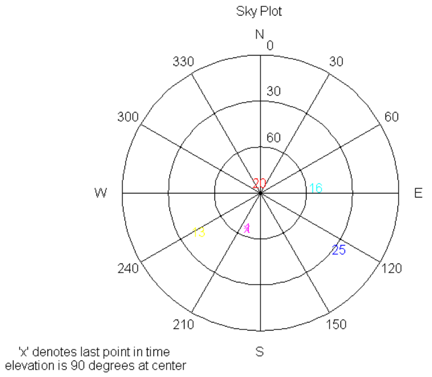


Figure 2 *The Sky plot of GPS Satellites*

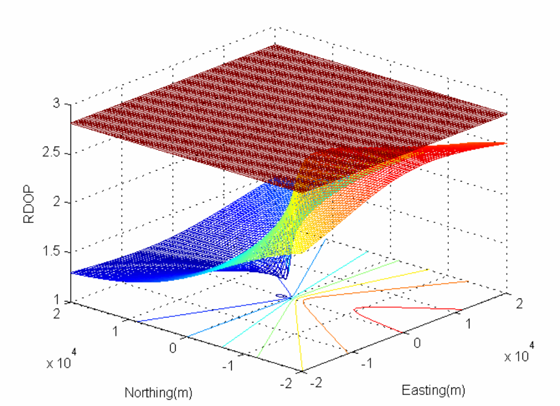


Figure 3 *RDOP changes with PL location*

The simulation results reveal how pseudolite location affects positioning geometry. GPS satellite distribution in Figure 2 was fixed at a certain time (UTC: 2004 3 22 0 33 0 at Sydney), and a single pseudolite location was varied from -20,000m to 20,000m horizontally with 500m intervals. The rover receiver is at the middle of the area and the altitudes of a mobile receive and the pseudolite was fixed at 500m and 5m respectively, which means the pseudolite elevation angle is varied through negative values. The upper mesh in Figure 3 shows the RDOP values without pseudolite augmentation and the lower one represents RDOP changes with the pseudolite location varying across the 500m grids. It is obvious that pseudolite significantly improves the satellite geometry. The RDOP values change between 1.2 and 2.8, depending on its location. Generally speaking, a pseudolite will strengthen the positioning geometry significantly if it is appropriately located.

In addition to analysing the pseudolite location effects, simulations were carried out to investigate the impact of the number of pseudolites used for positioning (Lee et al. 2002). Figure 4 shows the change of 24 hours' average values of RDOP, RHDOP, and RVDOP as a function of the number of pseudolites used. All the pseudolites' elevations were -17.5 degree.

We note that the value of RDOP declines from 3.2 to 1.5 with even one pseudolite, showing significant strengthening of the GPS-PL geometry. The employment of pseudolites whose elevation angle is below the horizontal makes the RVDOP values smaller than the RHDOP values, which is the reverse situation to GPS positioning without PL. The improvement obtained with more than two pseudolites is smaller than that with the inclusion of one or two pseudolites. Consequently only one pseudolite will be used in the following analysis of pseudolite augmentation.

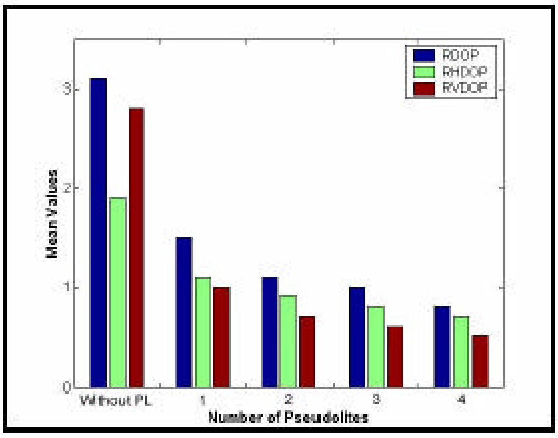


Figure 4 *Mean RDOP vs. number of pseudolites*

3. PSEUDOLITE OBSERVATION EQUATIONS

As with GPS satellite observations, pseudolite observation equations can be given as:

Pseudo Range: $R = r + c \cdot dt_r - c \cdot dt^p + d_{orb} + d_{trop} + d_{mR} + e_R$ (2)

Carrier Phase: $\Phi = r + c \cdot dt_r - c \cdot dt^p + d_{orb} + d_{trop} + l \cdot N + d_{m\Phi} + e_{\Phi}$ (3)

where R and F are code and carrier phase observables respectively; r is the geometric range between the PL and receiver; dt_r, dt^p, d_{orb} and d_{trop} are receiver and PL clock bias, pseudolite-location error and tropospheric delay respectively; d_{mR} and $d_{m\Phi}$ are multipath error for pseudo-range and carrier phase respectively; l and N are the wavelength and integer ambiguity; c is the speed of light; e_R and e_{Φ} are the pseudo-range and carrier phase observation noises, respectively.

Note that no terms need to be introduced to account for ionospheric delay with ground-based PLs. Nevertheless, the other error terms can be significant, and therefore it is inadvisable to use the PL signal for absolute positioning without correction. Instead, corrections should be done for the PL as well as for the GPS satellite measurements made by a ground reference station. The single-differenced pseudolite observables between two receivers can be expressed as:

Pseudo Range: $\Delta R = \Delta r + c \cdot \Delta dt_r + \Delta d_{orb} + \Delta d_{trop} + \Delta d_{mR} + \Delta e_R$ (4)

Carrier Phase: $\Delta\Phi = \Delta r + c \cdot \Delta dt_r + \Delta d_{orb} + \Delta d_{trop} + l \cdot \Delta N + \Delta d_{m\Phi} + \Delta e_{\Phi}$ (5)

The PL clock error is eliminated and the receiver clock error can be resolved by GPS observation. Previous analysis indicates that the noise levels for the PL are similar to those of the GPS (Choi et al. 2000). As the geometry to a PL cannot be considered identical for the mobile user and the reference station, orbit errors and troposphere delay can only be eliminated or reduced for the GPS satellite, but not for the PL. Therefore single differenced terms remain in the equations for PL location error and PL tropospheric error. The multipath term has been left in the equations because PL multipath seems to strongly affect the measurements. These three major errors, i.e. troposphere delay, PL location error and multipath, have to be considered carefully and will be addressed below.

4. PSEUDOLITE LOCATION ERROR

In GPS relative positioning, the impact of the orbit errors on baseline length was estimated approximately using the following ‘rule of thumb’ formula (Wells et al. 1987):

$$ {}^{db}\!/_{b} = {}^{dr}\!/_{r} \tag{6}$$

where db is the baseline error; dr is the orbit error; b is the baseline length; and r is the distance between satellites and users.

Equation (6) indicates that, in the case of short range GPS relative positioning, satellite orbit errors will have very little impact on the solutions. However, as pseudolites are close to users, the impact of the orbit errors needs more detailed analysis (Wang et al 2002).

Suppose there are the reference receiver A(0, 0 ,0), rover receiver B(a, b, h) and the pseudolite T(x, y, z), the vectors from PL antenna (T) to receivers A and B are $\overrightarrow{S_A}$ and $\overrightarrow{S_B}$ respectively, as shown in Figure 5. If dx , dy and dz are the errors in the PL coordinates x , y and z respectively (with dT is the norm of the errors), the impact of these errors on the single-differenced (SD) measurement between A and B is

$$dS_{AB} = d\left\|\overrightarrow{S_A}-\overrightarrow{S_B}\right\| = \left\|d\overrightarrow{S_{AB}}(x)+d\overrightarrow{S_{AB}}(y)+d\overrightarrow{S_{AB}}(z)\right\| \tag{7}$$

where

$$\begin{aligned} d\overrightarrow{S_{AB}}(x) &= \left(\frac{x}{S_A}-\frac{x-a}{S_B}\right)\cdot\overrightarrow{dx} \\ d\overrightarrow{S_{AB}}(y) &= \left(\frac{y}{S_A}-\frac{y-b}{S_B}\right)\cdot\overrightarrow{dy} \\ d\overrightarrow{S_{AB}}(z) &= \left(\frac{z}{S_A}-\frac{z-h}{S_B}\right)\cdot\overrightarrow{dz} \end{aligned} \tag{8}$$

The equations (7) and (8) indicate that the measurement error dS_{AB} is geometry-dependent, and the factor of dS_{AB}/ dT changes between 0 and 2 according to the A, B and T relative location.

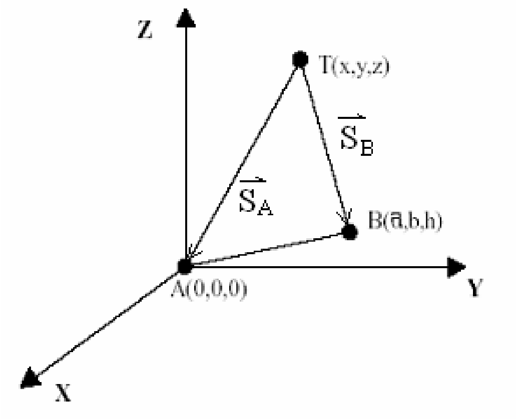


Figure 5 *Coordinates of reference receiver A, rover receiver B and the pseudolite* T

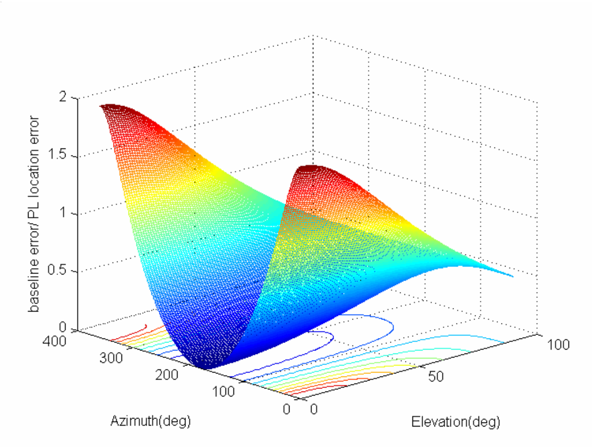


Figure 6 *Baseline error/PL location error when* $[x\ y\ z] = [0\ 100\ 0]$ *and* $[a_0\ b_0\ c_0] = [0\ 200\ 0]$

Figure 6 shows the numerical analysis results of the impact of PL location error on SD measurements. It can be seen clearly that the influence of PL location errors varies significantly with different elevation and azimuth of the rover receiver. In the worst case, the single-differencing procedure doubles the size of the pseudolite-location error in the measurements. Therefore, if the pseudolite is improperly located, the pseudolite-location errors can significantly bias the precise carrier phase observation even though the errors are only of the order of a few centimetres in magnitude.

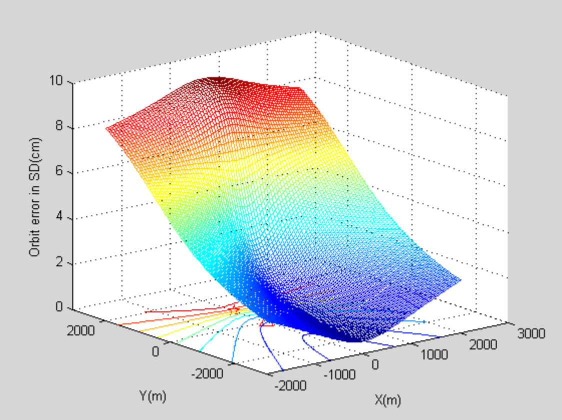


Figure 7 *Baseline errors when PL is inside the surveying area*

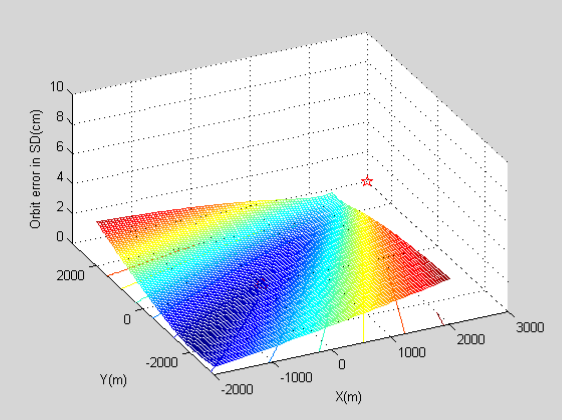
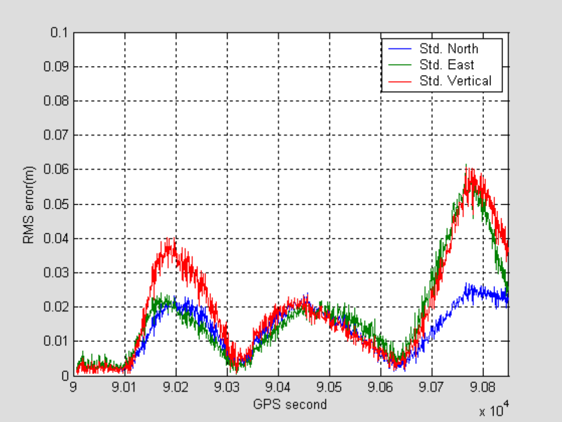


Figure 8 *Baseline errors when PL is outside the surveying area*

For analyzing the case in aerial photogrammetry, a simulation was conducted with one PL located inside (Figure 7) and outside (Figure 8) a 4km x 4km surveying area with the aircraft at 450 meters' flying height. The two figures show the baseline error measured by PL with 5 centimeter PL location errors in three directions. The SD error is much higher and more discrepant when the PL is located inside the airborne surveying area.



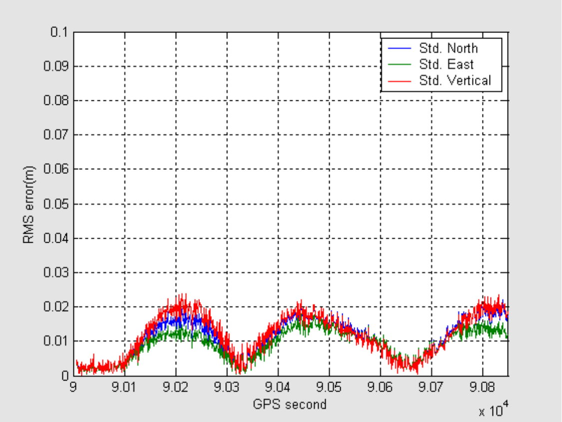


Figure 9 *RMS - PL inside surveying area*

Figure 10 *RMS - PL outside surveying area*

The measurements from one PL and five GPS satellites were included in the positioning solutions, and a comparison between the reference trajectory used for the measurement simulation and the resulting trajectory from the DD data processing was made to evaluate the positioning performance. A PL location error of 5cm in the 3 directions was included in the data processing. Figure 9 and 10 show the results of double differenced (DD) carrier phase measurements with one PL located at the positions shown in Figure 11. The mobile GPS receiver trajectory and the locations of the PL and reference receiver are shown in Figure 11.

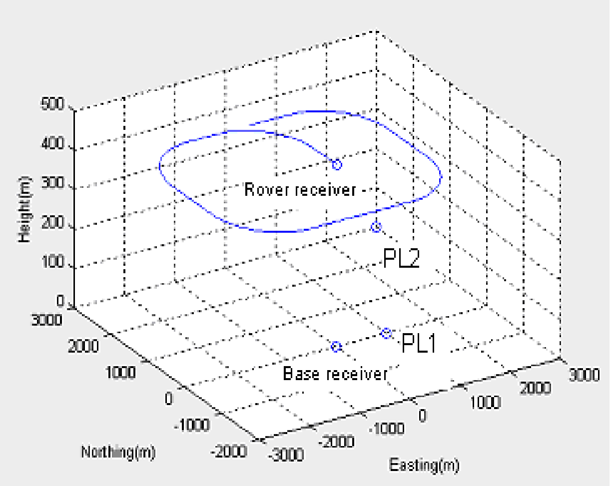


Figure 11 *3D mobile receiver trajectory and PL locations*

As shown in Figure 9 and 10, the root mean square (RMS) of the PL augmented GPS measurements is much larger when the PL is located inside surveying area (PL1 in Figure 11), than in the case when it is outside (PL2 in Figure 11). This result is consist with the formulas 7 and 8 derived above, and matches the simulation results of the SD pseudolite measurement error caused by PL location errors, as shown in Figure 7 and 8. Moreover, for airborne GPS positioning, even if the optimal PL location is selected, the PL location error cannot be neglected. Therefore pseudolite location should be precisely determined, using GPS surveying, total station or some other conventional surveying techniques.

It should be emphasised that equations (7) and (8) can be used for a thorough analysis of the effects of pseudolite location errors for any specific GPS/PL positioning application. The simulation studies have shown that the optimal locations of PL can minimise the impact of PL location errors on measurement models and on final positioning.

5. TROPOSPHERIC DELAY

Common tropospheric delay models, such as the Saastamoinen, Hopfield, or Black model, applied in standard DGPS processing work effectively for short baseline applications because the paths of GPS signal propagation are similar for both the user and reference receivers. However, the signal path from PL transmitting antenna to aircraft can be very different to the path to the reference receiver, and it changes rapidly as the aircraft moves. The differencing processing for PL data with the same tropospheric delay model is much less efficient than that for GPS data. Therefore, effective modelling of the troposphere effect has to be implemented for precise PL positioning. A few tropospheric delay models for PL measurement are introduced in this section and the impact of the modelling error on the PL measurement is analysed.

Weather conditions in lower troposphere tend to vary rapidly in both the vertical and the horizontal directions. While the latter variation cannot be efficiently modeled today, some models have been proposed to describe the altitude dependency.

RTCA defined standards on tropospheric delay models regarding Local Area Augmentation System with PL for aircraft landing (RTCA 2000). The tropospheric correction consists of a dry and a wet component.

$$\Delta_{trop} = \Delta_{dry} + \Delta_{wet} \tag{9}$$

The dry and the wet component are to be determined separately by the simplified model (10). The '*' in (10) is to be read as a parameter for dry, respectively wet.

$$\Delta_* = 10^{-6} \cdot N_* \cdot D_{rov} \cdot \left(1 - \frac{h_{rov} - h_{PL}}{h_{*,0}} \right) \tag{10}$$

The rover height h_{rov} and the PL height h_{PL} in (10) claim the importance of vertical distance for tropospheric modeling. D_{rov} is the slope distance between rover and PL. $h_{*,0}$ is a fixed scaled height for the model which is 42,700m for the dry component, and 13,000m for the wet component. These heights are empirically defined as the upper boundaries for dry and wet tropospheric refraction. Meteorological data are poured into the model to determine the refraction coefficient N_* which is defined by:

$$\begin{aligned} N_{dry} &= 77.6 \cdot \frac{P}{T} \quad \text{and} \\ N_{wet} &= 22770 \cdot \frac{f}{T^2} \cdot 10^{\frac{7.4475(T-273)}{T-38.3}} \end{aligned} \tag{11}$$

where T is the temperature in K, f is the relative humidity and P is the atmospheric pressure in mPa sampled on the spot. For the standard meteorological parameters (P=1013mPa, T=20°, f=50%), the dry and wet tropospheric delays can reach 268.3ppm and 50.98ppm respectively from Equation (11). The total tropospheric delay is 31.93cm per km. It is obvious that the magnitude of the tropospheric delay is too large to be ignored for precise PL positioning and the local weather conditions have a significant effect on the correction.

As the local refractivity should be estimated as a slowly varying parameter, an effective alternative to calculating N_* via a model is to estimate it using the PL measurements of the reference receiver (Barltrop et al. 1996). Different models are introduced (Biberger et al. 2003) and derived from the Hopfield model (Hofmann-Wellenhof et al 2000).

For airborne geo-referencing applications, if the reference receiver has the same height as the PL, the tropospheric delay after between-receivers single differencing can be derived from (10):

$$d\Delta_{trop} = 10^{-6}\cdot(N_{dry}+N_{wet})\cdot dr - D_{rov}\cdot(h_{rov}-h_{PL})\cdot\left(\frac{N_{dry}}{h_{dry,0}}+\frac{N_{wet}}{h_{wet,0}}\right) \qquad (12)$$

where dr is the difference in geometric ranges between the pseudolite transmitter and the two receivers. If the pseudolite and the reference receiver can be located with the dr and $D_{rov}\cdot(h_{rov}-h_{PL})$ as small as possible, the tropospheric error can be significantly mitigated. However, this may not be feasible in reality.

The impact of the SD tropospheric delay on the PL range measurement was simulated according to Equation (12). Figure 12 shows the result with the same surveying area, flight height and PL location as those for Figure 8. Assuming there is 1% error in the N_{dry} measurement and 10% in N_{wet}, the SD tropospheric error presented in Figure 13 is within a few centimeter level which changes with the aircraft position.

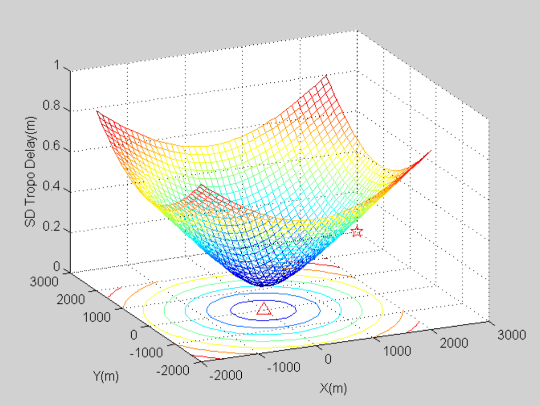


Figure 12 *SD tropospheric delay for PL changes with the rover receiver locations*

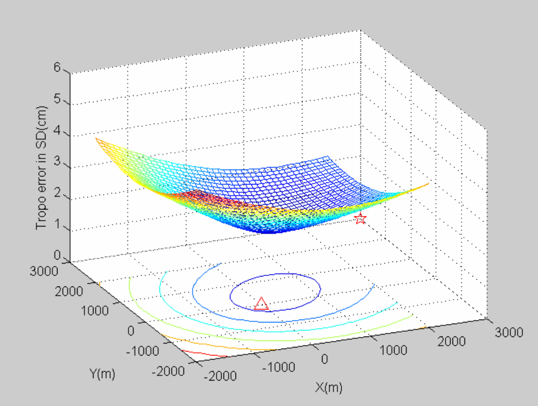


Figure 13 *Simulated SD tropospheric errors in a surveying area*

As the signal paths from the PL to the reference receiver and to the aircraft pass through very different parts of the troposphere, the DGPS principle cannot effectively mitigate tropospheric delay modeling error. Even more significant, the magnitude of the tropospheric delay is strongly dependent on vertical differences. It is necessary to optimize the tropospheric delay model for PL since the tropospheric delay error heavily effects airborne GPS positioning with PL augmentation. Further analysis on this will be carried out in a separate study.

6. MULTIPATH

Multipath will be present in both the code and carrier measurements if one or more reflected signals arrive at the receiver antenna in addition to the direct signal. There is no general model of the multipath effect because of the arbitrarily different geometric situations. The theoretical maximum multipath bias that can occur in pseudo-range data is approximately half a chip length of the code, 150m for C/A code ranges and 15m for the P(Y) code ranges. The carrier phase multipath does not exceed about one-quarter of the wavelength (5-6cm for L1 or L2).

There are several considerations to be made for PL multipath that are unique from GPS. As a PL emits a strong signal to the ground or nearby objects, reflections from them need to be considered. The primary means of mitigating PL multipath is to reduce the gain of the transmission towards the ground and nearby objects. This can be accomplished essentially by the use of a Multipath-Limiting-Antenna and avoiding locating the PL antenna close to building and other objects.

The PL multipath error on a ground reference receiver will be constant since both the PL and the reference receiver's antennas are stationary. Hence, the influence of multipath from PL cannot be mitigated and reduced to the same extent over time as in the case of GPS. However, because of the constant characteristics of the multipath from a pseudolite transmitter in a static environment, it is relatively easy to calibrate it in advance and remove it during data processing.

As with GPS, PL multipath is a challenging issue that needs to be solved for airborne positioning receiver. Good hardware design including receivers and receiver antennas, optimal antenna location on the aircraft, as well as software-based multipath mitigation techniques will be needed.

Multiple flight tests confirm that PL signals can be tracked with a top mounted antenna without a direct line of sight (Biberger et al. 2001). So there are two possibilities from which to choose. One is, as shown in Figure 1, to use a bottom-mounted antenna for PL reception only in addition to the top mounted GPS antenna. Another one is to use a top-mounted antenna for both GPS and PL. Using a top-mounted antenna only means that there is frequently no direct line-of sight between the PL and airborne antenna, which affects PL measurement quality, though some theoretical derivation and modeling has been conducted for signal creeping on the aircraft surface (Biberger et al. 2003). The signal creeping error is sensitive to a and ß in Figure 14. Using a second (bottom-mounted) antenna for the PL introduces the need for lever-arm corrections. This in turn necessitates the requirement for attitude determination in the user equipment as well as a more complicated positioning algorithm.

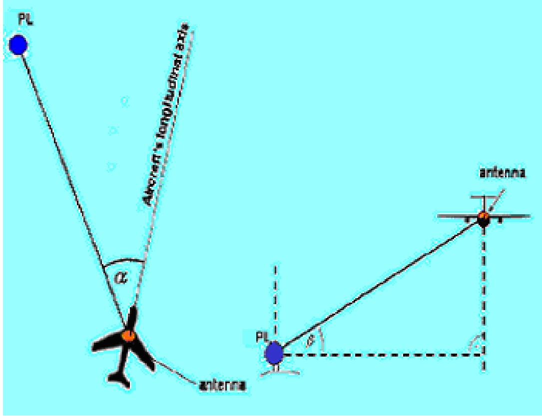


Figure 14 *Relative direction of PL*

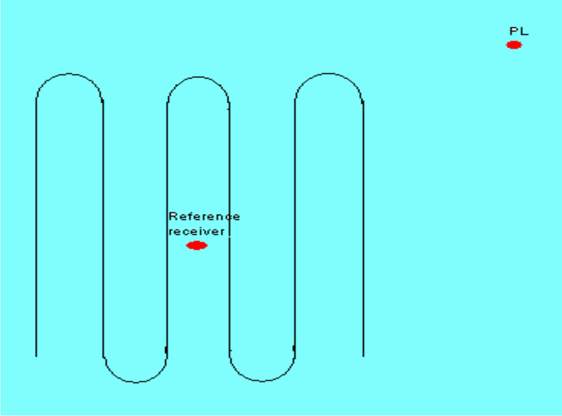


Figure 15 *Trajectory, PL and receiver location*

7. CONLUSIONS

The systematic error analysis and geometry design of PL augmented airborne GPS positioning have been discussed in this paper. Errors that affect the accuracy of PL measurements, such as PL location error, tropospheric delay modeling error and multipath have been analyzed in detail and error mitigation techniques have been proposed.

A comprehensive analysis should be conducted for the site design of airborne surveying. Locating the reference receiver at the centre of a surveying area and the PL outside it at proper direction and distance (as shown in Figure 15) can minimize impacts of PL location error and the tropospheric delay modeling error, and reduce the RDOP value at the same time. The pseudolite location should be optimally selected and precisely determined. An optimized tropospheric delay model is critical for accurate PL measuring and further research is needed into it, though the geometry design of PL and reference receiver also affects the measurement of troposphere error. PL multipath can be worse than that for GPS and signal creeping happens with top mounted antenna. In general, more effective modeling and compensating methods are expected to reduce corresponding errors in PL measurements.

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